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Controlling Combustion Phasing Variability with Fuel Injection Timing in a Multicylinder HCCI Engine

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***Abstract*— Reduction of combustion phasing cyclic variability (CV) in homogeneous charge compression ignition (HCCI) engines operating lean with late autoignition is experimentally demonstrated. A three–state discrete time model developed in [1] is used for controlling the fuel injection timing and is applied to a multicylinder engine. A key objective of this work is to reduce cyclic variability without advancing the mean combustion phasing. Specifically, if late combustion phasing can be made less variable without advancing the operating point then areas where high CV is typically encountered could be made less variable. Examples include load transition down, when the residual temperature drops more rapidly than can be manipulated by the valve timing, or during mode transitions.**

Experimental results are presented to gauge the effectiveness of two control schemes, namely proportional and state feedback which have been tuned using the three–state model. Each of these controllers have been augmented with an integrator to maintain the late combustion phasing requirement. This is also done to draw a fair comparison of the controllers ability to reduce CV. The controllers are tested at various levels of CV and it is found that simple control can reduce the standard deviation of combustion phasing an average of 17% over open loop behavior. In addition, because of the simplicity of the control, this offers a viable solution for commercial applications.

I. Introduction

Homogeneous charge compression ignition (HCCI) combustion offers gains in efficiency over traditional combustion concepts. However its high pressure rise rates and difficulty to control limit its use in commercial applications. One method to make HCCI more commercially viable is to run higher loads at later combustion phasings, effectively lowering the pressure rise rate of high load combustion to remain within the mechanical constraints of production engines. Similar to work done in [2], [3] where extension of the low load limit was the focus. While running higher loads at later phasings is a viable solution to the problem, another issue is encountered when combustion phasing is sufficiently late, namely cyclic variability (CV) in the form of oscillatory behavior [4], [5].

In addition, HCCI has high CV in many other regions of operation. During load transition, large gradients in temperature can induce variability [6]. Also since the operating region of HCCI is limited, and mode transitions are necessary, HCCI can be forced to operate in a region of high oscillations. If the variability in these regions can be suppressed then HCCI can become a more viable solution for reducing fuel consumption in a production vehicle.

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To mitigate these oscillations the model and control concepts developed in [1], [7] for lean, late phasing, high CV HCCI combustion with negative valve overlap (NVO) are applied here through experiments on a four cylinder engine. The model captures both the recycled thermal and chemical energy through the residual gas fraction from one cycle to the next and has been shown to work on both single and multicylinder engines. Specifically, start of injection (SOI) timing is utilized as the control input to the system with combustion phasing for feedback. The use of SOI for combustion phasing control has been applied through experiments previously in [8], [9] where the objective was mode transitions and load extension respectively. In addition, the use of SOI to reduce CV was presented in [10] while control of CV using valve timings on a cycle-to-cycle basis with a fully flexible valve train was performed in [11], [12]. A separate, but related problem is the control of CV in spark assisted compression ignition (SACI) combustion. This combustion mode typically runs at stoichiometry and exhibits different dynamic behavior than the lean HCCI case as shown through experiments in [13] and analysis with combustion models in [14], [15].

The key objective of this work is to evaluate the effectiveness of SOI control on reducing CV without shifting the mean phasing of the combustion. To do so the controllers presented here were augmented with an integrator to ensure that they were operating at the same mean combustion phasing as open loop, which is crucial for the evaluation as demonstrated later in Sec. IV. This was done so that one can see the reduction in CV while maintaining the late phasing associated with it.

In the following sections an explanation of the experimental set up will be given and an extension of the model from [1], [7] to accommodate a non-linear burn duration will be described. After that, controllers are designed using a linearization of the calibrated model and validated using experimental results.

II. Experimental Setup

A four cylinder 2.0 liter GM LNF Ecotec engine running premium grade indolene was used as the base line platform. Modifications to accommodate HCCI combustion include increasing the compression ratio to 11.25:1 and using camshafts with shorter duration and lower lift to allow for unthrottled operation. In addition to the stock turbo charger the engine was augmented with a small supercharger (Eaton M24) to provide boost. Results presented here were run at slightly boosted conditions, approximately 1.1 bar intake manifold pressure, λ =1.2, an engine speed of 1800 RPM and a load of

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approximately 3.25 bar net IMEP. The spark was left on, but at a position of 40° after top dead center. Since the mixture is lean and highly diluted with residuals the spark will have little influence on the combustion, having the spark on late only helps to prevents the spark plug from fouling.

Cylinder pressures were sampled at a resolution of 0.1 cad and to fully observe the complex dynamics of CV 3000 consecutive engine cycles were obtained for each test. In this study, different levels of CV were achieved by trapping progressively less and less residuals through a retarding of the exhaust valve closing (EVC) time.

The control strategies presented in this paper were implemented using a combination of C and Matlab code, and were tested in real-time using an ETAS ES910 rapid prototyping module. The module uses an 800 MHz Freescale PowerQUICC™ III MPC8548 processor with double precision floating point arithmetic and 512 MB of RAM.

III. Model

The model used for control development effectively captures the recycling of both thermal and chemical energy through recompression of residual gases, the model is explained in detail in [1]. A summary of the model is provided here along with a modification to accommodate the burn duration of a multicylinder engine. The model is discrete time and has three states: the temperature at intake valve closing, $T_{ivc}(k)$, the mass of fuel, $m_f(k)$ and a state to capture ignition delay, $A(k)$. The output of the model is combustion angle of 50% burned, $\theta_{50}(k)$, while residual gas fraction, x_r, and mass of injected fuel, m_i, are inputs. Fuel injection timing is a controlled input, $u(k)$.

The complete model is given by:

$$\begin{cases} T_{ivc}(k+1) &= f_1(x(k), x_r(k)) \\ m_f(k+1) &= f_2(x(k), x_r(k), m_i(k)) \\ A(k+1) &= f_3(u(k)) \end{cases} \tag{1}$$

where $x(k)$ and $u(k)$ are the state vector and controlled input respectively. The temperature dynamics are derived in [7] and can be summarized by

$$T_{ivc}(k+1) = (1 - x_r(k))T_{im} + x_r(k)T_r(k) \tag{2}$$

where $T_r(k) = g(x(k), x_r(k))$ is the residual gas temperature at the time of intake valve opening and T_{im} is the temperature of the fresh charge in the intake manifold. The mass of fuel state is used to determine the total mass of injected fuel in addition to any unburned fuel from previous cycles,

$$m_f(k+1) = m_i(k)+x_r(k)(1-\eta_m(k))(1-\eta_n)m_f(k). \tag{3}$$

The combustion efficiency during main compression (IVC $\to$ EVO), η_m, is modeled by

$$\eta_m(\theta_m(k)) = e_0\left[1 + \exp\frac{\theta_m(k) - e_1}{e_2}\right]^{-1}, \tag{4}$$

while the efficiency during the recompression region (EVC $\to$ IVO), η_n, is taken to be a constant. The end of combustion, θ_m, is dependent upon the start of combustion, θ_{soc}, and assumes an exponential burn duration

$$\begin{aligned}\theta_m(k) &= \theta_{soc}(k) + \Delta\theta(k) \\ &= \theta_{soc}(k) + d_0\exp\left(\frac{\theta_{soc}(k) - d_1}{d_2}\right).\end{aligned} \tag{5}$$

The burn duration differs from that presented in [1] where a linear relationship is used. The start of combustion is found from the temperature dynamics and the ignition delay state, $\theta_{soc}(k) = \theta_{soc}(T_{ivc}(k), A(k))$, where

$$A(k+1) = s_0 + s_1\left[1 + \exp\frac{u(k)}{s_2}\right]^{-1}. \tag{6}$$

The resulting output is the combustion phasing of 50% burned,

$$\theta_{50}(k) = \theta_{soc}(T_{ivc}(k), A(k)) + \frac{\Delta\theta(k)}{2}. \tag{7}$$

Model parameters have been determined from engine data run in open loop and at high CV on a per–cylinder basis.

IV. Control Development

This work focuses on reducing CV by control of the start of injection (SOI) during NVO. This actuator is chosen because it can be utilized on a cycle-by-cycle and cylinder-to-cylinder basis. Other possible actuators available for control are EVC timing and fuel quantity. The EVC timing is not ideal for this engine platform. The engine has stock hydraulic cam phasers which are slow, therefore cylinder-to-cylinder and cycle-to-cycle control is not possible. Fuel quantity is mainly used to track the desired load but can potentially be used for reducing CV. Preliminary experiments with control of both injection timing and quantity showed that the variability in combustion phasing was slightly reduced but, at the same time, the variability in the produced torque increased. For these reasons, fuel quantity was not controlled despite its high control authority.

Controllers for each cylinder are derived following the model-based design in [1] where proportional and state feedback controllers are developed based on linearizations and evaluated based on nonlinear simulation. The nominal point studied here is an engine speed of 1800 rpm, a load of 3.3 bar net IMEP corresponding to $m_i = 11\,\mathrm{mg/cycle}$, an average residual gas fraction $\bar{x}_r = 54\%$, and an injection timing $u_0 = 330°$. These values are well defined in an engine control unit except for the residual gas fraction, which is calculated using cylinder pressure data and can be described by random variations around the average value [13], [16]. In the design, the controller performance is therefore evaluated for a range of $\bar{x}_r$ around the nominal value. Table I shows how the eigenvalues of the linearized model move with varying $\bar{x}_r$ around the nominal speed and load.

A. Controller Evaluation

To convincingly demonstrate that the reduction in CV is due to control of SOI on a per-cycle basis, careful attention must be given to keep the same nominal conditions in both

TABLE I

LOCATIONS OF EIGENVALUES μ FOR THE LINEARIZED MODEL.

$\bar{x}_r$	49%	53%	54%	55%
μ_1	−4.1	−1.3	−0.87	−0.48
μ_2	−0.071	−0.067	−0.074	−0.098

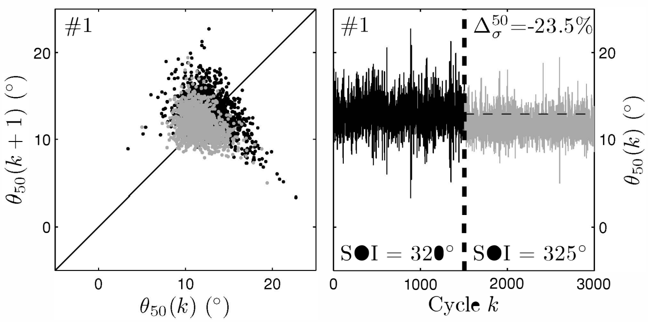

Fig. 1. Return maps (left) and time series (right) of combustion phasing as the result of a constant change in SOI. The aim here is to reduce CV in late phasing conditions when trivial steady-state solutions do not apply.

the open loop case and the closed loop case. Specifically, due to the high sensitivity at the edge of stability, it is imperative to maintain the combustion phasing close for the two cases. The controllers are therefore augmented with integral action, which is not done in past work [10]–[12]. To illustrate, Fig. 1 shows open loop data where the constant value SOI is set 5° earlier after half the experiment. This change advances the combustion phasing by 1.3° and reduces the standard deviation by 23.5%. It is typically trivial to reduce CV in steady state by phasing the combustion earlier through, e.g., earlier SOI, small increase in fuel mass, or earlier EVC. The reason for control of CV is when such steady state solutions do not apply such as, for example, if the combustion phasing is constrained to be late due to excessive pressure rise rates or during transients such as load transitions or combustion mode switches when the combustion traverses regions with high CV. Finally, for steady state evaluation, a large number of cycles should be recorded to accurately quantify the statistical properties. In this work 3000 consecutive engine cycles are recorded for each test, which is an order of magnitude more than is typical for engine dynamometer testing.

B. Proportional Control

Proportional control has been proposed to reduce oscillations in nonlinear systems with stochastic perturbations in the parameters [17]. For the system studied here, the nonlinear dynamics in (1) are governed by the thermal and the chemical coupling between cycles and there are random variations of the residual gas fraction [16]. The proportional controller is

$$\delta u(k) = K_p(\theta_{50}(k) - \theta^*_{50}) \tag{8}$$

where θ^*_{50} is the desired reference combustion phasing. To find K_p, the eigenvalues of the linearized closed-loop model, given by Eq. (1) and (8), are studied for a range of K_p [1]. From the root locus of eigenvalues the gain $K_p = 1.0$ was chosen in order to stabilize the system and minimize

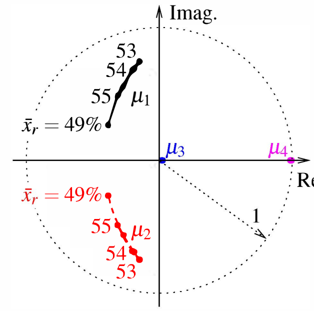

Fig. 2. The root locus with proportional control shows oscillatory but stable closed loop dynamics for a range of residual gas fraction $\bar{x}_r$. The open-loop eigenvalues in this range are shown in Tab. I.

oscillatory behavior. Integral action is introduced by adding the term

$$K_i \sum_k (\theta_{50}(k) - \theta^*_{50})T(k) \tag{9}$$

to the controller (8) where $T(k) = 120/N(k)$ is the time for one engine cycle at the speed N rpm. The gain K_i is chosen to slowly reach steady state with a time constant of circa 100 cycles while not changing the transient response.

The performance for varying average residual gas fraction $\bar{x}_r$ is evaluated by computing the closed loop eigenvalues of the linearized model, Fig. 2 shows the result for cylinder 1. The value of θ^*_{50} is set corresponding to the nominal point for the open loop model. For $\bar{x}_r$ between 49% and 55%, the closed loop dynamics are oscillatory but (marginally) stable. Outside this range, the authority of SOI saturates and θ^*_{50} is not reached. The loci of the other cylinders suggests similar behavior.

C. State Feedback Control

To improve the transient response over proportional control, an LQG design was performed based on the linearized model. The Kalman filter is designed by setting the noise variance to 1 and the state covariance matrix diagonal with the elements 100, 1, and 10. For the regulator, the criterion $\sum_{k=1}^{\infty}\left(10\delta\theta_{50}(k)^2 + \delta u(k)^2\right)$ is minimized. The resulting regulator is composed of two lag compensators and one pole and zero that nearly cancel and is thus reduced to one lag compensator without any appreciable change in the magnitude and phase for all frequencies of the closed loop system. The lag compensator stabilizes the system and to obtain zero steady-state error integral action is introduced by Eq. (9), which did not have a significant influence on the transient response. The final controller is

$$C(z) = -1.13\frac{(z-1.04)(z+0.054)}{(z-1)(z-0.49)}. \tag{10}$$

Analogous to Fig. 2, the closed loop dynamics of the controller for cylinder 1 (the other cylinders exhibit similar behavior) are shown in Fig. 3 for varying $\bar{x}_r$. The eigenvalue locations translate to oscillatory responses that are more damped than with proportional control if $\bar{x}_r$ is 50% or above.

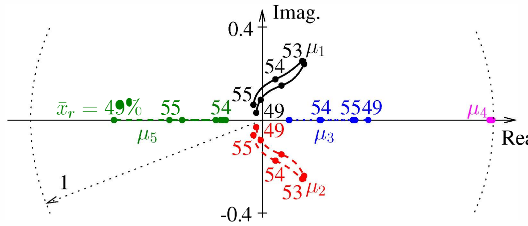

Fig. 3. The root locus with state feedback control for a range of residual gas fraction $\bar{x}_r$. The open-loop eigenvalues in this range are shown in Tab. I

V. Experimental Results

To test the effectiveness of the controllers described in the preceding sections they were implemented through rapid prototyping hardware coupled with the engine instrumentation, actuators and its engine control unit (ECU). The ECU calculated combustion phasing in real time based on net heat release analysis from the cylinder pressure transducers. This was then used for feedback.

The objectives of the tests were to demonstrate reduction of CV and to make comparison of the effectiveness of different control schemes on various levels of CV. To minimize variations in experiments due to environmental conditions the tests presented here were performed consecutively allowing a fair comparison of the results to be drawn. However, similar relative improvements were observed when repeating the tests on a day to day basis. The tests were run in open loop for approximately the first 1500 cycles while controllers were active for the remaining 1500 cycles.

A. Comparison of Controllers

Each controller was run at three different EVC positions, the result being three values of residual gas fraction and CV. The residual gas fractions span a range of approximately 1% and correspond to the values in Figs. 2, 3. A summary of the results can be found in Table II. This table presents the mean combustion phasing for open and closed loop, $\bar{\theta}_{50}^{OL}$, $\bar{\theta}_{50}^{CL}$, as well as the open loop standard deviation, σ_{50}^{OL}, and the percent reduction of this standard deviation in closed loop, Δ_{σ}^{50}. For example, Table II for cylinder 1 and EVC=102.5° indicates that the percent reduction in standard deviation of θ_{50} from open loop using proportional control was Δ_{σ}^{50} =21.4%.

Figure 4 shows the results of cylinders 1 and 2 in Table II through the use of return maps. A return map shows the relationship between consecutive cycles which provides insight on the dynamic behavior of the CV. Here the return map is the plot of the combustion phasing in cycle k versus the combustion phasing in cycle $k+1$. When variability is low the phasing in cycle k should deviate minimally from the phasing in cycle $k+1$ and therefore the phasing always returns to a similar value. More simply, as a process becomes less variable the return map contracts closer to the diagonal. In Fig. 4 the open loop cycles are in grey and closed loop in black. One can observe that the controlled cycles have less dispersion in early and late events indicating that the control was effective. The model was parameterized for the middle case, corresponding to a residual gas fraction of 51% and as

TABLE II
Summary of proportional control (left) and state feedback control (right) of four cylinders with three levels of CV.

	Proportional				**State Feedback**			
	EVC = 102.5° aTDC							
Cyl	$\bar{\theta}_{50}^{OL}$	$\bar{\theta}_{50}^{CL}$	σ_{50}^{OL}	Δ_{σ}^{50}	$\bar{\theta}_{50}^{OL}$	$\bar{\theta}_{50}^{CL}$	σ_{50}^{OL}	Δ_{σ}^{50}
1	11.0	10.5	1.9	-21.4	10.5	10.5	1.7	-4.7
2	10.8	10.5	2.1	-24.8	10.2	10.5	1.6	-0.3
3	10.8	10.5	1.9	-10.6	10.4	10.5	1.7	-0.5
4	10.3	10.5	1.6	-7.0	9.8	10.5	1.4	-12.1
	EVC = 101.5° aTDC							
1	11.6	11.0	2.1	-19.5	11.8	11.0	2.1	-11.9
2	11.1	11.0	2.2	-22.3	11.2	11.0	2.3	-19.4
3	11.2	11.0	2.2	-16.8	11.5	11.0	2.2	-17.3
4	11.1	11.0	2.0	-15.3	11.3	11.0	2.1	-20.2
	EVC = 100.5° aTDC							
1	12.2	11.8	2.6	-24.0	11.9	11.8	2.4	-16.4
2	11.7	11.8	3.2	-17.6	11.8	11.8	3.0	-15.8
3	11.8	11.8	2.6	-13.6	11.8	11.8	2.6	-1.2
4	11.6	11.8	2.4	-12.5	11.3	11.8	2.1	-4.3

expected the controllers perform best when at this operating point. Proportional control is more robust to deviations from this point, this can be seen in the reduction in standard deviation in Table II. One can also observe that the integrator has effectively controlled the mean combustion phasing to a set point and that the difference from open loop is small. The results of Table II show that both control methods were effective in reducing the CV in all cases. However, some were better than others. The small reductions at low CV are most likely due to the fact that there is little room for improvement, the process is not exhibiting a large increase in combustion phasing dispersion. In the high CV case there is more potential for a decrease in dispersion, however as the CV increases the process becomes more complex. Since linear control is used it is reasonable that performance is limited in narrow regions around the linearization.

B. Evaluation of Control Effectiveness

Because the mean combustion phasing from open loop has been maintained in closed loop the effectiveness of the control is more easily observed through a percent reduction of standard deviation. Figure 5 shows this through the time series data of combustion phasing and the control signal, SOI, for all four cylinders. The results shown here are for the state feedback control on the medium level of CV. One can see that there is a visible reduction in the variability of the combustion when transitioning from open loop (grey) to closed loop (black). In addition the SOI perturbations are small, less than ±5°, and the reduction is achieved while maintaining the perturbation around the open loop SOI value.

Return maps and symbolic statistics for this same data are displayed in Fig. 6. Symbolic statistics is a non-linear time series technique used to quantify the probability that specific sequences of cycles may occur, for a detailed explanation of this tool see [18], [19]. Here the sequence length is set to 3 and the times series data is divided into 5 equally

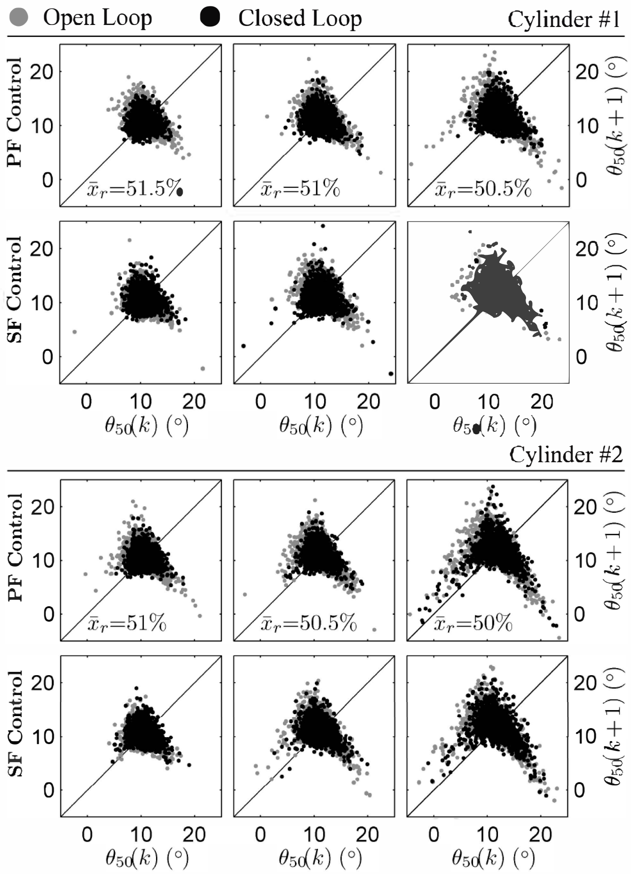

Fig. 4. Comparison of return maps of combustion phasing for proportional and state feedback control. Open loop cycles are in grey and closed loop in black. From left to right the tests have a progressively smaller x_r resulting in an increasing level of CV. The least and most variable cylinders are shown.

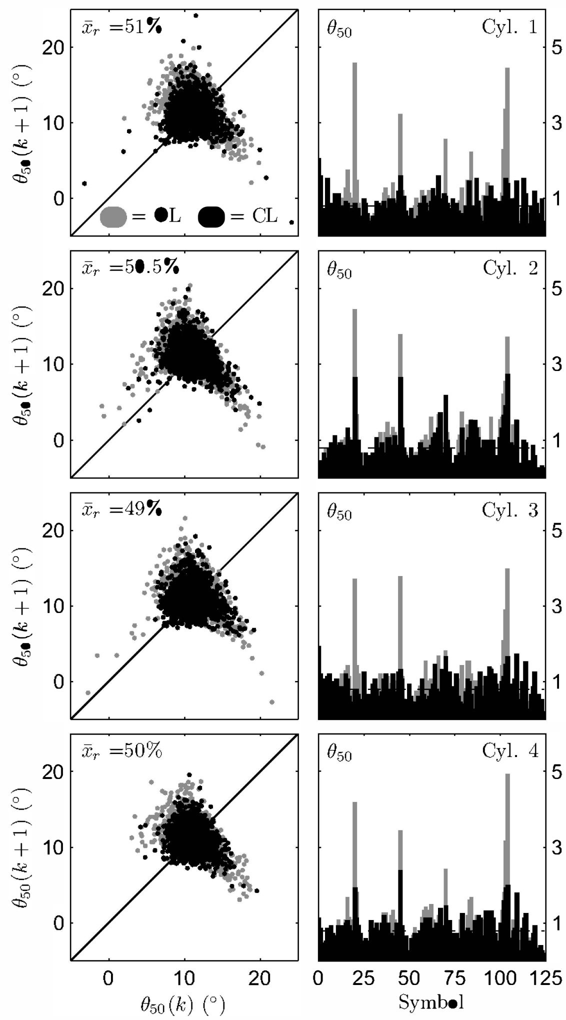

Fig. 6. Return maps and symbolic statistics of state feedback control at medium CV. Open loop results in grey and closed loop in black. The contraction of the return maps and flattening of the symbolic statistics indicate effective control.

probable bins. Since the bins are divided to be equally probable any combination of bins is equally probable if there are no couplings between cycles, as is the case for highly stable combustion. The result of this would be a flat relative frequency spectrum. However, as we transition into high variability we observe spikes in the frequency spectrum indicating that some sequences are more likely to occur than others and that the dynamics are deterministic, which are predicted by the model as shown in [1]. It is therefore desired to not only contract the return map close to the diagonal using control but to also flatten the spectrum in the symbolic statistics. In addition if the return maps and symbolic statistics were featureless then all that would be left in the system is noise, no deterministic coupling between cycles, and further reduction would require more complex control.

Figure 6 clearly shows that control achieves both of these goals for all four cylinders despite the noted difference in residual gas fraction in each cylinder. Results for other cases were similar and are not shown to conserve space.

VI. Conclusions

It has been shown through experiments that model based control developed in [1] using fuel injection timing can effectively reduce the cyclic variability observed in late phasing HCCI operating at lean and lightly boosted conditions. Multiple levels of CV have been tested and a reduction in standard deviation of combustion phasing was realized in all cases. However, it was observed that proportional control, which was tuned based on the model, is more robust to different levels of CV in comparison to state feedback. The state feedback controller at the nominal point did out perform the proportional controller.

The addition of an integrator to the proportional and state feedback control allowed for the mean combustion phasing to be maintained from the open loop case allowing a fair comparison of different controllers and their effectiveness at reducing CV to be evaluated. The maximum amount of reduction in CV was found to be 25% while the average reduction was 17%. This is comparable to the reduction obtained by simply shifting the mean phasing to a significantly earlier value. However, because the operating point was not shifted in the control presented here, this indicates that simple control could be used to reduce the CV in unavoidable areas, such as during load transitions and during mode switches.

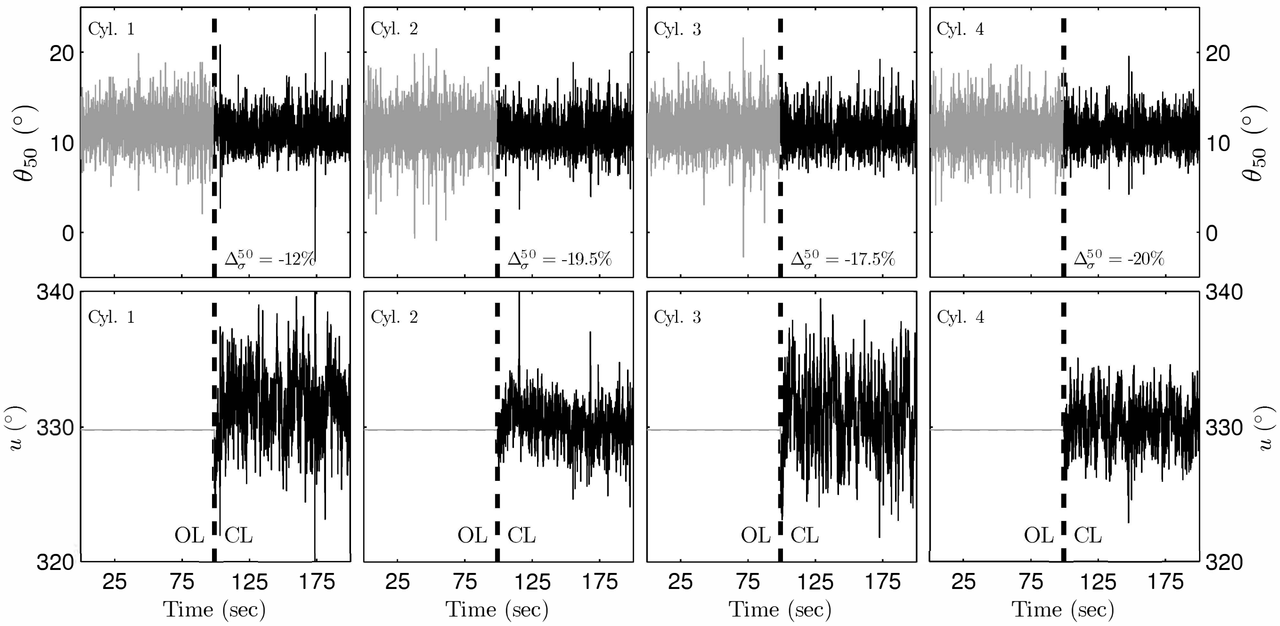

Fig. 5. Time series data of state feedback control at medium CV. Open loop results in grey and closed loop in black show the reduction in combustion phasing variability (top) and the SOI control signal (bottom).

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