Speed Gradient Approach to Longitudinal Control of Heavy-Duty Vehicles Equipped with Variable Compression Brake

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Abstract

Speed regulation of Heavy-Duty Vehicles equipped with variable compression brake is considered in this paper. Use of compression brake reduces the wear of the conventional friction brakes, and it is, thus, a preferred way of controlling the vehicle speed during a steady descent or non-critical braking maneuvers. To perform more aggressive (critical) braking maneuvers or control vehicle speed during large changes in the grade, the compression brake must be coordinated with gear ratio adjustments and friction brakes. In this paper we develop nonlinear controllers that accomplish both critical and non-critical maneuvers as well as in-traffic vehicle following objectives. The design technique is based on the Speed-Gradient approach, whereby control action is selected in the maximum descent direction for a scalar goal function. The nominal goal function is selected to address the speed regulation objective and, then, is appropriately modified by barrier functions to capture constraints due to complimentary driving objectives.

1 Introduction

The last ten years have witnessed a significant increase in the efficiency and operational speed of the Heavy-Duty Vehicle (HDV) powertrains. It is ironic that while increased fuel efficiency results in high acceleration performance, it also reduces the vehicle natural retarding capability, and hence, limits the deceleration performance of HDVs. The main vehicle retarders, namely, the friction brakes have well known limitation associated with overheating (Gerdes et al., 1995), saturation, and actuator delays (Yanakiev and Kanellakopoulos, 1997). The current practice of "snubbing" rather than "dragging" the service brakes exemplifies these limitations (Fitch, 1994). Operational speeds comperable to passenger vehicles, where safe braking can be achieved, require high retarding power with consistent magnitude and unlimited duration. Thus, augmenting the braking performance of HDVs with auxiliary retarding mechanisms is increasingly important in order to integrate HDVs in advanced transit and highway systems. Indeed, vehicle manufacturers aggressively develop retarding mechanisms with low weight and maintenance requirement so they do not offset the recent improvements in powertrain efficiency.

A retarding mechanism that satisfies the above requirements is the engine compression brake. During compression

braking mode the engine dissipates the vehicle kinetic energy through the work done by the pistons to compress the air during the compression stroke. The compressed air is consequently released into the exhaust manifold through a secondary opening of the exhaust valve at the end of the compression stroke. We call the secondary opening of the exhaust valve as Brake Valve Opening (BVO). Due to geometric constraints, the valve lift profile is considerably different for the exhaust and brake events (see Figure 1). In conventional compression braking mechanisms, BVO is fixed with respect to the crank angle degrees resulting in on-off retarding mechanisms (Cummins, 1966). Selective activation of the BVO in a number of cylinders can provide discretely variable retarding power (Jacobs Vehicle Systems, 1999). The retarding mechanism we consider here allows continuously variable retarding power through control of BVO (Hu et al., 1997). The timing of BVO (specified in crank angle degrees) is the input signal to the compression braking mechanism and is physically limited to the range $u_{cb}^{min} = 620$ to $u_{cb}^{max} = 680$ degrees after Top Dead Center (TDC) as shown in Figure 1.



Figure 1: Lift profiles for exhaust, intake and brake events.

In this paper we concentrate on the longitudinal control problem using compression braking to its maximum extent in an effort to minimize the conventional friction brake usage and, hence, the friction brake wear. It is well known that wear and overheating reduces the DC authority of the friction brakes and introduces large parameter variations. Adaptive algorithms have been developed by Ioannou and Xu (1994) to address unpredictable changes in brake model parameters. Recent work by Maciuca and Hedrick (1998) shows that non-smooth estimation and adaptation techniques can be used to achieve a reasonable brake friction force control. The delays associated with the pneumatic activation of friction brakes impose one of the main obstacles in autonomous heavy vehicle following scenarios. These difficulties in autonomous HDVs are mitigated by using aggressive prediction algorithms (Yanakiev and Kanellakopoulos, 1997). The prediction algorithms, however, assume accurate knowledge of the delays and do not perform well during a totally uncertain brake maneuver. To reduce the application and intensity of the

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friction brakes, compression brake can potentially be used as a sole decelerating actuator during low deceleration requests (i.e., non-critical braking maneuvers) and combined with the friction brakes during high deceleration requests (i.e., critical braking maneuvers).

In particular, we consider an automatic vehicle speed control problem, namely, the regulation of vehicle speed during a long descent down a grade. To sustain the desired vehicle speed during non-critical maneuvers such as a steady descent, we use compression brake only. To perform more aggressive (critical) maneuvers or control vehicle speed during large changes in the grade, the compression brake must be coordinated with gear ratio adjustments and friction brakes to supplement the compression braking capability. The control design is based on a reduced-order nonlinear approximation of the crankangle-based engine model developed in (Moklegaard *et al.*, 2000). The braking torque due to compression is a nonlinear function of the timing of BVO and the engine (i.e., vehicle) speed. This nonlinear dependence introduces additional difficulties on the actuator level.

We develop nonlinear controllers that accomplish both critical and non-critical maneuvers as well as in-traffic vehicle following objectives. The controllers are designed using the Speed-Gradient (SG) methodology (Fradkov, 1979; Fradkov and Pogromsky, 1999). This is a general technique for controlling nonlinear systems through an appropriate selection and minimization of the goal function. The nominal goal function is selected to address the speed regulation objective and, then, is appropriately modified by barrier functions to capture constraints due to complimentary driving objectives. The controller is designed to provide the decrease of the goal function along the trajectories of the system. The local closed-loop stability is verified analytically by checking the achievability condition. It is shown that the controller has a large region of attraction covering a very reasonable interval of initial values for the vehicle speed.

The paper is organized as follows. The model for longitudinal vehicle speed is described in Section 2. In Section 3 we review the necessary results of the Speed Gradient methodology. In Section 4 we develop a Speed-Gradient algorithm for speed control using only compression brake and demonstrate the controller performance during small changes in the grade. For large changes in the grade the compression brake must be coordinated with gear ratio adjustments and an appropriate controller for doing this is also described in Section 4. The coordination with friction brake is described in Section 5. In Sections 6,7 we address critical maneuvers, in particular aggressive braking and "vehicle-following" are considered. The closed-loop performance for all traffic scenarios is demonstrated through simulations.

2 Vehicle Dynamics Model

Consider the vehicle operation during a driving maneuver on a descending grade with β degrees inclination ($\beta = 0$ corresponds to no inclination, $\beta < 0$ corresponds to a descending grade). It is assumed that during the descent, the engine is not fueled and is operated in the compression braking mode. A lumped parameter model approximation is used to describe the vehicle longitudinal dynamics during compression braking. For fixed gear operation the engine crankshaft rotational speed, ω , is expressed by:

$$J_t \dot{\omega} = T_{cb} + r_g (F_\beta - F_{qdr} + F_{fb}) \tag{1}$$

where,

 ω is the engine rotational speed, (rad/sec), related to the vehicle speed value, v, (m/sec), by the following relation

$$v = \omega r_g, \tag{2}$$

 $r_g = \frac{r_{\omega}}{g_t g_{fd}}$ is the total gear ratio, where r_{ω} is the wheel diameter, g_t is the transmission gear ratio, g_{fd} is the final drive gear ratio (assumed to be known)

 $J_t = mr_g^2 + J_e$ is the total vehicle inertia reflected to the engine shaft (depends on the vehicle loading conditions), where J_e is the engine crankshaft inertia

m is the mass of the vehicle (depends on the mass of payload), (kg)

 $F_{qdr} = C_q v^2 = C_q r_g^2 \omega^2$ is the quadratic resistive force (primarily, force due to aerodynamic resistance, but we also include friction resistive terms)

 $C_q = \frac{C_d A \rho}{2} + C_f$ is the quadratic resistive coefficient, where C_d is the aerodynamic drag coefficient, ρ is ambient airdensity, *A* is the frontal area of the vehicle, C_f is the friction coefficient (assumed to be known)

 $F_{\beta}(m,\beta)$ is the force due to road grade (β) and the rolling resistance of the road(μ):

$$F_{\beta}(m,\beta) = -\mu gm\cos\beta - mg\sin\beta$$

g is the acceleration due to gravity

 F_{fb} is the force on the vehicle due to application of the conventional friction brake (negative during friction braking)

 T_{cb} is the shaft torque applied by the engine to the driveshaft (negative during compression braking).

The speed control problem is to ensure that the vehicle speed v(t) tracks the desired reference vehicle speed v_d as the truck proceeds the descending grade: $v \rightarrow v_d$. Since the engine rotational speed $\omega(t)$ is related to the vehicle speed by $v = \omega r_g$, this ensures that $\omega \rightarrow \omega_d$ where $\omega_d = \frac{v_d}{r_g}$ is the desired engine speed. Additionally, we assume that the braking with the compression brake is preferable, because we want to preserve the friction brake and use the friction brake only when absolutely necessary.

The desired controller is designed using the Speed-Gradient (SG) methodology (Fradkov and Pogromsky, 1999) reviewed in Section 3. This is a general technique for controlling nonlinear systems through an appropriate selection and minimization of the goal function. The goal function Q is selected to address the speed regulation objective, i.e.

$$Q = \frac{J_t \gamma_0}{2} (v - v_d)^2 \ge 0, \quad \gamma_0 > 0.$$
 (3)

The controller is designed to provide the convergence to zero of the goal function (3) along the trajectories of the system (1) that implies the achievement of the speed regulation problem $v \rightarrow v_d$. Taking into account the relation between engine and vehicle speeds (2), the goal function can be rewritten as follows:

$$Q = \frac{J_t \gamma}{2} (\omega - \omega_d)^2 \ge 0, \quad \gamma = \gamma_0 / r_g^2 > 0. \tag{4}$$

3 Speed-Gradient Methodology

In this section we review the necessary results of the Speed-Gradient (SG) Control Methodology (Fradkov, 1979; Fradkov and Pogromsky, 1999). Consider a nonlinear system model of the form

$$\dot{x} = f(x) + g(x)u, \tag{5}$$

where $x \in \mathbb{R}^n$ is the state vector, $u \in \mathbb{R}^m$ is the control input vector, f(x) and g(x) are continuously differentiable vector-functions.

The control design objective is to stabilize a desired equilibrium $x = x_d$ (that satisfies $f(x_d) + g(x_d)u_d = 0$) while at the same time shaping the transient response via the minimization of the following scalar goal function

$$Q(x(t)) \to 0$$
, when $t \to \infty$, (6)

where Q(x) is assumed to be twice continuously differentiable and radially unbounded function that satisfies $Q(x) \ge 0$, $Q(x_d) = 0$. The function Q may, for example, represent a weighted sum of the squares of the deviations of the different components of x from the corresponding components of x_d . We first present an intuitive argument leading to the derivation of the SG controller. Consider the evolution of Q(x(t))over a sufficiently small time interval $[t, t + \Delta t]$. Then, the objective of minimizing Q can be restated as

$$Q(t + \Delta t) \approx Q(t) + \omega(x(t), u(t))\Delta t \rightarrow \min,$$

where the function $\omega(x, u)$ is determined as a time derivative of Q(x) along the trajectories of the system (5) (i.e. the speed of change of Q):

$$\omega(x,u) = \dot{Q} = \frac{\partial Q}{\partial x}(f(x) + g(x)u).$$

To prevent large control excursions from the desired steadystate value, u_d , we can augment a control penalty and consider the minimization of the function

$$Q(t) + \omega(x(t), u)\Delta t + \frac{1}{2}(u - u_d)^{\mathrm{T}} (\frac{\Pi}{\Delta t})^{-1}(u - u_d) \to \min_{u}, \ \Pi > 0$$
(7)

Since $\omega(x(t), u)$ is affine in *u* the minimizer is obtained by setting the gradient with respect to *u* to zero. This leads to the controller

$$u(t) = u_d - \Pi \Psi(x), \tag{8}$$

where Ψ is the gradient of the "speed" $\dot{Q} = \omega(x(t), u)$ with respect to *u*:

$$\Psi(x) \stackrel{\Delta}{=} \nabla_u \omega(x(t), u) = \left(\frac{\partial Q}{\partial x}g(x)\right)^{\mathrm{T}}.$$
(9)

This controller is referred to as the Speed-Gradient Proportional (SG-P) controller. One can also augment a penalty on the control increment and consider the minimization of

$$Q(t) + \omega(x(t), u)\Delta t + \frac{1}{2}(u(t) - u(t - \Delta t))^{\mathrm{T}}\Gamma^{-1}(u(t) - u(t - \Delta t)) \to \min_{u(t)}, \ \Gamma > 0.$$
(10)

This results in the Speed-Gradient Integral (SG-I) controller:

$$\dot{u}(t) \approx \frac{u(t) - u(t - \Delta t)}{\Delta t} = -\Gamma \nabla_u \omega(x(t), u) = -\Gamma \Psi(x).$$
(11)

The general class of controllers of interest for this paper are Speed-Gradient Proportional plus Integral (SG-PI) controllers of the form:

$$u(t) = u_d - \Pi \Psi(x(t)) - \Gamma \int_0^t \Psi(x(s)) ds.$$
 (12)

In general, there is no guarantee that the controller results in the stable closed loop system and is robust to disturbances. However, one may provide some stability and robustness properties under some additional assumptions. Rewrite the control law (12) in a more convenient equivalent form:

$$u = u_d - \Pi \Psi(x) + \theta, \quad \dot{\theta} = -\Gamma \Psi(x)$$

where θ is the integrator state. Let us consider the following Lyapunov function

$$V(x,\theta) = Q(x) + \frac{1}{2}\theta^{\mathrm{T}}\Gamma^{-1}\theta \ge 0$$
(13)

and calculate its time-derivative along the trajectories of the closed loop system (5), (12):

$$\dot{V} = \frac{\partial Q}{\partial x} (f(x) + g(x)u_d) - \Psi^{\mathrm{T}}(x)\Pi\Psi(x).$$
(14)

Now, let us determine the following sets:

$$\Upsilon_C \stackrel{\Delta}{=} \{x : Q(x) \le C\}, \quad \Omega_C \stackrel{\Delta}{=} \{(x, \theta) : V(x, \theta) \le C\},\$$

and suppose that the so called *achievability condition* holds:

$$\frac{\partial Q}{\partial x}(f(x) + g(x)u_d) \le -\rho(Q(x)) \quad \text{for all } x \in \Upsilon_C, \quad (15)$$

where ρ is a continuously differentiable function that satisfies $\rho(0) = 0$, $\rho(z) > 0$ if $z \neq 0$. Since the achievability condition holds for $x(t) \in \Upsilon_C$, then $\dot{V}(t) \leq 0$ as long as $x(t) \in \Upsilon_C$. Assume that the initial condition at time t = 0 is $(x(0), \theta(0))$ such that $(x(0), \theta(0)) \in \Omega_C$, i.e. $x(0), \theta(0)$ satisfy the following inequalities:

$$Q(x(0)) \le C \cdot \lambda, \quad \frac{1}{2} \theta(0)^{\mathrm{T}} \Gamma^{-1} \theta(0) \le C \cdot (1-\lambda), \quad 0 \le \lambda \le 1.$$

Then, for all t, $V(x(t), \theta(t)) \leq V(x(0), \theta(0)) \leq C$ and $Q(x(t)) \leq C$ so that the achievability condition holds on the trajectory $x(t, x(0), \theta(0))$, $\theta(t, x(0), \theta(0))$. Therefore,

 $V(x(t), \theta(t) \text{ and } Q(x(t)) \text{ are bounded and the closed-loop sys$ $tem trajectories <math>x(t), \theta(t)$ are bounded as well due to radial unboundness of Q(x). Then from Barbalat's lemma, we obtain that $Q(x(t)) \to 0$ as $t \to \infty$. Additionally, assuming that $Q(x_d) = 0, Q(x) > 0$ for $x \neq x_d$, we get $x \to x_d$. Hence, the above facts prove the following result.

Theorem 1: Consider the SG-PI controller (12) applied to the system (5). Assume that the achievability condition (15) holds for all $x \in \Upsilon_C$. Then for all initial conditions $(x(0), \theta(0))$ in Ω_C the closed loop trajectories satisfy

$$\lim_{t\to\infty}Q(x(t))=0$$

Moreover, if Q(x) satisfies $Q(x_d) = 0$ and Q(x) > 0 for $x \neq x_d$, the closed loop system meets the control objective $\lim_{t\to\infty} x(t) = x_d$.

Remark 1: The set $\Omega_C = \{(x, \theta) : Q(x) + \frac{1}{2}\theta^T\Gamma^{-1}\theta \le C\}$ describes the region of attraction of the equilibrium $(x_d, 0)$. Typically, $\theta(0)$ is set to zero, and then all initial states x(0) in $\Upsilon_C = \{x : Q(x) \le C\}$ are guaranteed to be recoverable by the controller (12).

Remark 2: The same result can be proved for the SG-P controller (8). Indeed, in this case the Lyapunov function *V* coincides with the objective function Q(x) and the region of attraction is the set Υ_C .

The vector u_d in the SG-P controller (8) and SG-PI controller (12) can be interpreted as an ideal feedforward term: $f(x_d) + g(x_d)u_d = 0$. Due to plant parameter variations, u_d may be unknown. However, in the case of the SG-PI controller (since the controller employs an integral action), we expect some robustness properties to disturbances that are additive to the plant input. Let *w* be an unknown constant additive disturbance affecting the plant input. Using *w* to represent the error in the feedforward term, the controller then can be viewed as applying an erroneous feedforward \tilde{u}_d in the form $\tilde{u}_d = u_d + w$. Thus, the SG-PI controller can be represented as

$$u = u_d - \Pi \Psi(x) + \theta + w, \quad \dot{\theta} = -\Gamma \Psi(x), \tag{16}$$

where θ can be interpreted as an estimate of -w. The desirable property $\theta(t) \rightarrow -w$, and, therefore, $\tilde{u}_d + \theta(t) \rightarrow u_d$ means that the integrator state corrects for the error in the feedforward asymptotically.

Consider the following Lyapunov function

$$\tilde{V}(x,\theta) = Q(x) + \frac{1}{2}(\theta+w)^{\mathrm{T}}\Gamma^{-1}(\theta+w)$$
(17)

and determine the set:

$$\tilde{\Omega}_C \stackrel{\Delta}{=} \{ (x, \theta) : \tilde{V}(x, \theta) \le C \}.$$

Theorem 2: Consider the SG-PI controller (16) applied to the system (5). Assume that the achievability condition (15) holds for all $x \in \Upsilon_C$. Then for all the initial conditions $(x(0), \theta(0))$ in $\tilde{\Omega}_C$ the closed loop trajectories satisfy: $\lim_{t\to\infty} Q(x(t)) = 0$. Moreover, if Q(x) satisfies $Q(x_d) = 0$ and Q(x) > 0 for $x \neq x_d$, the closed-loop system meets the control objective $\lim_{t\to\infty} x(t) = x_d$. If, furthermore, the $n \times m$ matrix $g(x_d)$ has a full column rank, then $\lim_{t\to\infty} \theta(t) = -w$, $\lim_{t\to\infty} (\tilde{u}_d + \theta(t)) = u_d$.

Remark 3: The set $\tilde{\Omega}_C = \{(x,\theta) : Q(x) + \frac{1}{2}(\theta + w)^T \Gamma^{-1}(\theta + w) \leq C\}$ describes the set of initial conditions for which the closed loop system trajectories meet the control objective (6). Although it is advantageous to have an initial estimate of -w, $\theta(0)$, as close as possible to -w, we typically set $\theta(0)$ to zero, because *w* is unknown. Then, the set of initial states x(0) that are guaranteed to be recoverable by the controller (12), decreases when *w* increases.

Remark 4: To check the achievability condition (15) the following procedure is used. Assume that $\Upsilon_C = \{x : Q(x) \le C\}$ for some C > 0 is a compact set with x_d in its interior and Q(x) > 0 if $x \ne x_d$, $Q(x_d) = 0$. We need to find a value of \tilde{C} such that for all $x \in \Upsilon_{\tilde{C}} = \{x : Q(x) \le \tilde{C}\}$ the strong achievability condition

$$\dot{Q}(x,u_d) = \frac{\partial Q}{\partial x}(f(x) + g(x)u_d) \le -\varepsilon Q(x), \quad (18)$$

where $\varepsilon > 0$, holds. Essentially, ε is a low bound on a rate of convergence of Q(x) to zero on the trajectories of the open-loop system. Let us define the following function

$$\kappa(C) \stackrel{\Delta}{=} \max_{Q(x) \le C} \left(\frac{\dot{Q}(x, u_d)}{Q(x)} \right). \tag{19}$$

Note that $\kappa(C)$, in general, may take an infinite value since $Q(x_d) = 0$. On the other hand, $\dot{Q}(x_d, u_d) = 0$ and, hence, $\frac{\dot{Q}}{Q}$ may have a removable singularity at 0 and we can, therefore, set $\dot{Q}(x_d, u_d)/Q(x_d) = \lim_{\|x\|\to x_d} \frac{\dot{Q}(x, u_d)}{Q(x)}$. In this case $\kappa(C)$ takes a finite value due to the compactness of Υ_C . The case that $\frac{\dot{Q}}{Q}$ has a removable singularity at x_d is, actually, rather usual in many applications. Moreover, $\kappa(C)$ is non-decreasing in *C*. The value of $\kappa(C)$ can be calculated using numerical optimization. From the graph of $\kappa(C)$ we may be able to specify $\tilde{C} > 0$ such that $\kappa(C) < 0$ for all $C \leq \tilde{C}$. Then, $\dot{Q}(x, u_d) \leq -\varepsilon Q(x)$ as long as $Q(x) \leq \tilde{C}$, i.e., the strong achievability condition (18) holds.

4 Speed control using only compression brake

Consider the system with compression brake only

$$J_t \dot{\omega} = T_{cb} + r_g (-F_{qdr} + F_\beta). \tag{20}$$

In (Moklegaard *et al.*, 2000) we develop a crank-angle based engine model that captures the effects of the compression braking. As a result of model reduction it is shown in (Moklegaard *et al.*, 2000a) that the compression brake torque on the crankshaft T_{cb} can be calculated using a static nonlinear function of the engine speed ω and the timing of BVO u_{cb} :

$$T_{cb}(\omega, u_{cb}) = \alpha_0 + \alpha_1 \omega + \alpha_2 u_{cb} + \alpha_3 u_{cb} \omega.$$
(21)

The timing of brake valve opening is limited to the range $u_{cb}^{min} = 620$ to $u_{cb}^{max} = 680$ degrees. These translate into the limits on the torque $T_{cb}^{min}(\omega) = T_{cb}(u_{cb}^{max}, \omega), T_{cb}^{max}(\omega) =$

 $T_{cb}(u_{cb}^{min}, \omega)$. Then the system under consideration looks as follows:

$$U_t \dot{\omega} = \alpha_0 + \alpha_1 \omega + \alpha_2 u_{cb} + \alpha_3 \omega u_{cb} + r_g (-F_{qdr} + F_\beta), \quad (22)$$

In accordance with SG method, we first calculate a time derivative of the goal function $Q = \frac{J_t \gamma}{2} (\omega - \omega_d)^2 \ge 0$, where $\omega_d = v_d/r_g$, along the trajectories of (22):

$$\dot{Q} = \gamma(\omega - \omega_d) \left(\alpha_0 + \alpha_1 \omega + \alpha_2 u_{cb} + \alpha_3 u_{cb} \omega + r_g (-F_{qdr} + F_\beta) \right)$$

and the derivative of \dot{Q} with respect to u_{cb} ("speed-gradient"):

$$\nabla_{u_{cb}}\dot{Q}=\gamma(\omega-\omega_d)(\alpha_2+\alpha_3\omega).$$

Then the SG-PI control law looks as follows:

$$u_{cb} = u_d - k_p \nabla_{u_{cb}} \dot{Q}(\omega) - k_i \int_0^t \nabla_{u_{cb}} \dot{Q}(\omega) ds \qquad (23)$$

where u_d is the feedforward of desired value for the input:

$$u_d = \frac{r_g(C_q v_d^2 - F_\beta) - \alpha_0 - \alpha_1 \omega_d}{\alpha_2 + \alpha_3 \omega_d}.$$
 (24)

Note that (23) can be interpreted as traditional PI controller but with *nonlinear* gains which depend on engine speed ω . As shown in Section 3, the implementation of the SG-PI controller (23) is possible without knowing precisely the value of the feedforward term u_d , due to the integral term.

The verification of the closed-loop stability requirements is done in accordance with the procedure in Remark 4 (see Section 3). According to the procedure let us consider a set

$$\Upsilon_C = \{ \omega : Q(\omega) \le C \} = \{ \omega : (\omega - \omega_d)^2 \le \frac{2C}{J_t \gamma} \}$$

for some C > 0 and then specify a value of \tilde{C} such that for all $\omega \in \Upsilon_{\tilde{C}}$ the achievability condition $\dot{Q}(\omega, u_d) \leq -\varepsilon Q(\omega)$, where $\varepsilon > 0$, holds. Calculating the ratio $\frac{\dot{Q}(\omega, u_d)}{Q(\omega)}$ and taking into account the expression (24) for the feedforward u_d , we get:

$$\frac{\dot{Q}(\omega, u_d)}{Q(\omega)} = -\frac{2C_q r_g^3}{J_t}(\omega + D).$$
(25)

It can be verified numerically that $D = \omega_d - \frac{\alpha_3 u_d + \alpha_1}{C_q r_g^3}$ is always positive for all physically feasible values of the grade β , mass *m* and desired engine speed ω_d .

Note that (25) reaches its maximum value on the compact set Υ_C at $\omega = \omega_d - \sqrt{\frac{2C}{L_V}}$, i.e.,

$$\max_{\Upsilon_C} \left(\frac{\dot{Q}(\omega, u_d)}{Q(\omega)} \right) = -\frac{2C_q r_g^3}{J_t} (\omega_d - \sqrt{\frac{2C}{J_t \gamma}} + D).$$
(26)

Then, we can guarantee that the achievability condition always holds for all $\omega \in \Upsilon_{\tilde{C}} = \{\omega : Q(\omega) \leq \tilde{C}\}$, where \tilde{C} is any positive number such that

$$\tilde{C} < \frac{J_t \gamma}{2} (\omega_d + D)^2.$$

In particular, since D > 0, we can select $\tilde{C} = \frac{J_t \gamma}{2} \omega_d^2$. Then, the set of initial states $\omega(0)$ in $\Upsilon_{\tilde{C}} = \{\omega : (\omega - \omega_d)^2 \le \omega_d^2\}$.

is guaranteed to be recoverable by the controller (23) with $\theta(0) = 0$ provided that w = 0. This implies that the controller (23) has a large region of attraction covering a very reasonable interval of initial values for the vehicle speed that corresponds to the engine speed interval of $[0, 2\omega_d]$, where $\omega_d = v_d/r_g$ is the desired engine speed.

We tested through simulations the operation of the SG-PI controller during a non-critical maneuver, when only compression braking is used to sustain the desired vehicle speed during a long descent. The vehicle mass is 20,000 kg, and the value of desired vehicle speed $v_d = 8.78$ m/sec (or 31.6 km/h) corresponds to desired engine speed $\omega_d = 1500$ rpm in the gear number seven. Figures 2, 3 illustrate the SG-PI controller response to unmeasured changes in road grade. The implementation of the controller is done with a value of the feedforward term u_d calculated assuming a grade of $\beta = 2.5$ deg while the actual grade changes from 1.8 to 4.2 degrees. The unknown grade creates an unmeasured disturbance which is additive to the control input. As shown in Theorem 2 (Section 3), the SG-PI controller ensures robustness properties to such kind of disturbances since the controller has an integral state which corrects the error in the feedforward u_d . The compression brake is used as the sole decelerating actuator, i.e., without activating friction brakes. It can be seen that although the timing of BVO, u_{cb} , saturates during the transients the antiwindup compensation that we used in combination with our controller preserves good speed regulation performance.



Figure 2: Controller responses to disturbance in road grade from 1.8 to 4.2 degrees: trajectories of grade, engine speed and vehicle speed. The desired engine and vehicle speeds are shown by the dashed line.

Since the braking torque is limited, in steady-state the compression brake can only support a certain range of vehicle speeds, v_d (or ω_d), for a given grade, β . Or, stated differently, given the desired vehicle velocity, v_d , we can only drive down a hill of a grade that falls within a certain range. To calculate this range, consider the steady-state balance of forces (or torques):

$$-T_{cb}/r_g + F_{qdr}(v, r_g) = F_{\beta}(m, \beta).$$

Given desired velocity v_d , gear ratio r_g and vehicle mass m, the determination of feasible grade range β_{min} , β_{max} is an elementary root-finding problem:

$$-\mu gm\cos\beta_{min} - mg\sin\beta_{min} = -T_{cb}^{max}(v_d)r_g^{-1} + C_q v_d^2,$$



Figure 3: Controller responses to disturbance in road grade from 1.8 to 4.2 degrees: trajectories of BVO timing and compression torque.

$$-\mu gm\cos\beta_{max} - mg\sin\beta_{max} = -T_{cb}^{min}(v_d)r_g^{-1} + C_q v_d^2.$$

In the driving scenario, shown in Figure 2, the feasible values for the road grade are within the range $\beta_{min} = 1.62$ degrees, $\beta_{max} = 4.37$ degrees. Therefore, for given vehicle mass and gear ratio the resulting compression brake is capable to support the desired speed v_d during the maneuver on a descending grade with inclination from 1.8 to 4.2 degrees. However, if we operated on a grade β that exceeds the maximum value β_{max} , the compression brake would not be able to support the desired velocity v_d under the same values of the mass and gear ratio. In this case we need to switch the gear number to a lower one (downshift) in order to increase the braking capability. The gear switching can be done by following rule: we downshift from the gear number k to the gear number k - 1 if the timing of BVO u_{cb} saturates, (i.e. $u_{cb} = u_{cb}^{max}$) and the speed fails to decrease, i.e., $\dot{\omega} > 0$. If the gear (k-1) is not sufficient (i.e, still $u_{cb} = u_{cb}^{max}$ and $\dot{\omega} > 0$) we downshift to gear number (k-2), etc. Note that in this scenario it can happen that there exists no gear ratio which would be able to guarantee the desired speed v_d for given grade β . In this case we need to activate the friction brake to supplement the lack of compression braking capability. A similar procedure is used for the upshifting based on the condition $u_{cb} = u_{cb}^{min}$ and $\dot{\omega} < 0$.

Figures 4, 5 illustrate the driving maneuver on a descending grade which changes from 1.8 to 7 deg. The value of the desired vehicle speed is $v_d = 8.78$ m/sec (or 31.6 km/h). The switch from the gear number seven to the gear number six takes place at t = 10 seconds. The value of desired vehicle speed $v_d = 8.78$ m/sec (or 31.6 km/h) corresponds to desired engine speed $\omega_d = 1500$ rpm in the gear number seven and $\omega_d = 1955$ rpm in the gear number six.

5 Coordination with friction brake

As can be seen from Section 4, the compression braking torque can be potentially used as the sole decelerating actuator at all potential gears without the assistance of friction brakes during non-critical maneuvers. Although we concentrate on speed control using only the compression brake, our approach can be extended to coordinate the compression brake with the friction brakes when it is necessary (i.e., during aggressive or critical maneuvers).

The conventional friction brake force on the wheel F_{fb}



Figure 4: Controller responses to a disturbance due to road grade change from 1.8 to 7 degrees: trajectories of grade, engine speed, vehicle speed, gear ratio. The desired engine and vehicle speed are shown by the dashed lines.



Figure 5: Controller responses to disturbance in road grade from 1.8 to 7 degrees: trajectories of BVO timing and compression torque.

can be considered as a static nonlinear and uncertain function of the pneumatic friction brake actuator temperature T and the control signal u_{fb} :

$$F_{fb} = f_{fb}(T, u_{fb}).$$

Recall that the braking with the compression brake is preferable, because we want to preserve the friction brake. Hence, we use the friction brake only when absolutely necessary. Specifically, if u_{cb} saturates, (i.e., $u_{cb} > u_{cb}^{max}$ or $u_{cb} < u_{cb}^{min}$) we calculate the torque deficit

$$\Delta T Q_{cb} = T Q_{cb}(\omega, u_{cb}) - T Q_{cb}(\omega, sat(u_{cb}))$$

and deliver it with the friction brake, $F_{fb} = \frac{\Delta T Q_{cb}}{r_g}$. Having made this convention, it is sufficient to consider the compression brake only with the idea that any extra braking effort required will be supplemented by the friction brake, according to the expression that we gave.

6 Speed control during aggressive braking maneuvers

Consider again the system (22). An additional objective is to ensure aggressive braking maneuvers when the difference between the current vehicle velocity, v, and the desired one, v_d , is sufficiently large, i.e., when $|v - v_d|$ does become greater then a given number $\varepsilon_1 > 0$. Assuming that the gear ratio r_g remains constant, the aggressive braking is needed when $|\omega - \omega_d| \ge \varepsilon$, where $\varepsilon = \varepsilon_1/r_g$, $\omega_d = v_d/r_g$. To capture the new requirement, the new objective function Q_1 has to include the nominal objective function $Q = \frac{J_t \gamma}{2} (\omega - \omega_d)^2$ and a smooth barrier function ϕ_1 which is zero when $|\omega - \omega_d|$ is smaller than ε and is monotonically and rapidly increasing when $|\omega - \omega_d|$ is larger than ε :

$$Q_1 = \frac{J_t \gamma}{2} (\omega - \omega_d)^2 + \frac{J_t \gamma_1}{3} \phi_1 (\omega - \omega_d) \ge 0, \quad \gamma_1 > 0,$$

where (see Figure 6)

$$\phi_1(\omega - \omega_d) = \begin{cases} 0, & \text{if } \omega - \omega_d \le \varepsilon \\ (\omega - \omega_d - \varepsilon)^3, & \text{if } \omega - \omega_d > \varepsilon \\ -(\omega - \omega_d + \varepsilon)^3, & \text{if } \omega - \omega_d < -\varepsilon \end{cases}$$



Figure 6: Barrier function for aggressive braking maneuver

In this case the SG controller looks as follows:

$$u_{cb} = u_d - k_p \nabla_{u_{cb}} \dot{Q}_1 - k_i \int_0^t \nabla_{u_{cb}} \dot{Q}_1(\omega(s)) ds,$$

where $\nabla_{u_{cb}} \dot{Q}_1 =$

$$\left\{ \begin{array}{l} \gamma(\omega - \omega_d)(\alpha_2 + \alpha_3\omega), & \text{if } \omega - \omega_d \le \varepsilon \\ \left(\gamma(\omega - \omega_d) + \gamma_1(\omega - \omega_d - \varepsilon)^2\right)(\alpha_2 + \alpha_3\omega), \text{if } \omega - \omega_d > \varepsilon \\ \left(\gamma(\omega - \omega_d) - \gamma_1(\omega - \omega_d + \varepsilon)^2\right)(\alpha_2 + \alpha_3\omega), \text{if } \omega - \omega_d < -\varepsilon \end{array} \right.$$

It means that if the $|\omega - \omega_d|$ falls outside the acceptable range $[-\varepsilon \ \varepsilon]$ then ϕ_1 takes a large value and forces the controller to respond rapidly. Thus, this control design ensures that normally the speed control is accomplished with the compression brake only but if we need to brake suddenly the barrier function amplifies the braking action and potentially causes the friction brake to engage. In this critical maneuver both the compression brake and friction brake are coordinated to decelerate rapidly.

Figures 7, 8 illustrate the critical driving scenario with aggressive braking. The value of $\varepsilon_1 = 0.29$ m/sec (or 1.05 km/h) corresponds to $\varepsilon = 50$ rpm in the gear number seven. In Figure 7 we compare the engine and vehicle speed during aggressive control action with the engine and vehicle speed during nominal control action. As can be seen, the response of the controller with the barrier function is much faster than that of the nominal design. Note the curvature change in the vehicle speed trajectory at about t = 0.3 sec. when the barrier function action vanishes as the engine speed is sufficiently close to the desired value.



Figure 7: The engine and vehicle speed during aggressive control action (solid lines) and nominal control action (dashed lines). The desired engine and vehicle speed are shown by dash-dotted lines.



Figure 8: BVO timing and friction force during aggressive control action (solid lines) and nominal control action (dashed lines).

7 Speed control within traffic

We next study a problem where our vehicle follows a leading vehicle (also a truck). This is an important automatized driving scenario in Automated Highway Systems (AHS) (see Shladover et al., 1991; Ioannou and Chien, 1993; Chen and Tomizuka, 1995; Yanakiev and Kanellakopoulos, 1996). We want to avoid any collisions between our vehicle and the leading truck. It means that we want to ensure that there is a sufficient distance between our vehicle and the vehicle in front of our vehicle. Let *s* be the position of our truck as it goes down the hill, so that $\dot{s} = v$, and s_l be the position of the leading vehicle as it goes down the hill.

The objective is then to always ensure that the separation distance (in seconds of travel) does not fall below a given number $\delta_1 \ge 0$

$$\frac{\delta_l - s}{v} \ge \delta_1. \tag{27}$$

As in Section 6, here we assume that the gear ratio r_g remains constant. Therefore, the objective (27) can be restated as $\frac{s_l-s}{\omega} \ge \delta$, where $\delta = \delta_1/r_g$ and the new objective function Q_2 , which captures the new requirement (27) will include the nominal objective function $Q = \frac{J_l \gamma}{2} (\omega - \omega_d)^2$ and a smooth barrier function ϕ_2 that penalizes the small headway between the trucks in seconds, i.e.,

$$Q_2 = \frac{J_t \gamma}{2} (\omega - \omega_d)^2 + J_t \gamma_2 \phi_2(\frac{s_l - s}{\omega}) \ge 0, \quad \gamma_2 > 0,$$

where ϕ_2 has to be zero when $\left|\frac{s_l-s}{\omega}\right|$ is larger than δ and mono-

tonically and rapidly increasing when $\left|\frac{s_l-s}{\omega}\right|$ is smaller than δ .

Because of $s - s_l < 0$ (since our truck follows the leading vehicle), the function ϕ_2 can be introduced as follows:

$$\phi_2(x) = \begin{cases} -1 - \frac{\delta}{x} & \text{if } x < -\delta \\ 0 & \text{otherwise,} \end{cases}$$

where δ is the minimum headway distance allowed between the trucks (see Figure 9).



Figure 9: Barrier function for "vehicle-following" maneuver.

Then the SG-PI controller has the following form:

$$u_{cb} = u_d - k_p \nabla_{u_{cb}} \dot{Q}_2 - k_i \int_0^t \nabla_{u_{cb}} \dot{Q}_2(\omega(s)) ds,$$

where

$$\nabla_{u_{cb}}\dot{Q} = \begin{cases} (\gamma(\omega - \omega_d) - \gamma_2 \frac{\delta}{s - s_l})(\alpha_2 + \alpha_3 \omega) & \text{if } \frac{s - s_l}{\omega} < -\delta \\ \gamma(\omega - \omega_d)(\alpha_2 + \alpha_3 \omega) & \text{otherwise} \end{cases}$$

This control design ensures that normally the speed control is accomplished with the compression brake but if $\left|\frac{s_I-s}{\omega}\right|$ becomes smaller than δ , a high gain braking action is produced and both the compression brake and friction brake are engaged to prevent the collision.

The idea of the simulation scenario is that the lead vehicle decelerates to $0.5v_d$ at t = 5 seconds and then accelerates again to v_d at t = 10 seconds. The minimum distance is $\delta = 10$ seconds (corresponding to $\delta_1 = 0.56$) is allowed. The responses are shown in Figures 10, 11. Note the aggressive braking action that the controller uses to prevent the collision with a decelerating leading vehicle.



Figure 10: The engine speed, vehicle speed, vehicle position during vehicle-following maneuver (solid lines). The dashdotted line shows the desired engine and vehicle speeds while the dashed lines show the vehicle and position trajectory of the leading vehicle.



Figure 11: BVO timing and friction force during vehicle-following maneuver.

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