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QUANTIFYING CYCLIC VARIABILITY IN A MULTI-CYLINDER HCCI ENGINE WITH HIGH RESIDUALS

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ABSTRACT

Cyclic variability (CV) in lean HCCI combustion at the limits of operation is a known phenomenon, and this work aims at investigating the dominant effects for the cycle evolution at these conditions in a multi-cylinder engine. Experiments are performed in a four-cylinder engine at the operating limits at late phasing of lean HCCI operation with negative valve overlap (nvo). A combustion analysis method that estimates the unburned fuel mass on a per-cycle basis is applied on both main combustion and the nvo period revealing and quantifying the dominant effects for the cycle evolution at high CV. The interpretation of the results and comparisons with data from a single-cylinder engine indicate that, at high CV, the evolution of combustion phasing is dominated by low-order deterministic couplings similar to the single-cylinder behavior. Variations, such as in air flow and wall temperature, between cylinders strongly influence the level of CV but the evolution of the combustion phasing is governed by the interactions between engine cycles of the individual cylinders.

NOMENCLATURE

m_f	Total in-cylinder fuel mass.
m_i	Injected fuel mass.
m_u	Unburned fuel mass.
ivo,ivc	Intake valve opening/closing.
evo,evc	Exhaust valve opening/closing.
nvo	Negative valve overlap (evc–ivo).
m, n	Sub/superscripts denoting the main combustion period (ivc–evo) and the nvo period (evc–ivo) respectively.
$\theta_{50}^m, \theta_{50}^n$	Combustion phasing for main/nvo period.
Q_m, Q_n	Gross heat release for main/nvo period.
η_m, η_n	Combustion efficiency for main/nvo period.

INTRODUCTION

For favorable operating conditions, homogeneous charge compression ignition (HCCI) combustion is typically more fuel-efficient while producing less emissions compared to spark-ignited or compression-ignited combustion. The operating range for HCCI is however limited and observed behavior indicates non-linear feedback between engine cycles that can stabilize or destabilize the combustion process. Specifically, low cyclic variability (CV) is observed in the favorable range [1] while oscillations in combustion phasing with high CV at late phasing [2, 3] and irregular phenomena at higher loads also have been reported [4].

HCCI is achieved in this work by a negative valve overlap (nvo) strategy where large amounts of the residual gases are trapped and re-compressed through an early closing of the exhaust valve [5]. The focus here is on the CV exhibited in lean conditions at late phasing. High CV in HCCI with nvo at stoichiometric conditions with spark-assist has been analyzed in [6, 7] and shows qualitatively different dynamic behavior than the lean case. CV in HCCI achieved by heating the intake air is analyzed in [8] and the influence of different fuels is examined in [9, 10]. Such strategies have a very low amount of residual gases compared to operating with nvo, which affects the feedback between cycles and modifies the combustion behavior. Stability analysis of HCCI with large amounts of residual gas is done in [11, 12] where instabilities and sensitivity in phasing are attributed to the thermal dynamics. In [13] an extended two-state model is derived, including the fuel mass dynamics, that predicts the observed behavior at high CV of lean HCCI in a single-cylinder engine.

Experiments and analysis with a re-compression strategy at conditions with notable CV are reported in [14] for a single-cylinder engine and for multi-cylinder engines in [15, 16]. These works examine the operating limits at different operating condi-

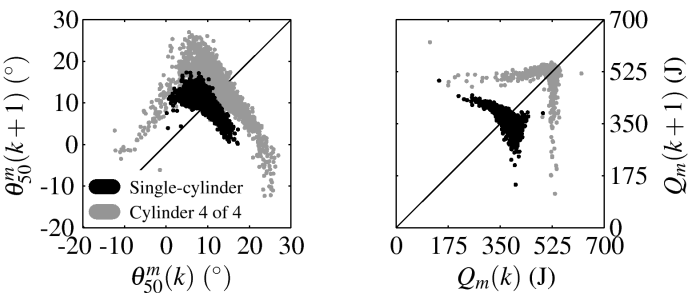

FIGURE 1. Return maps for combustion phasing θ_{50}^m and gross heat release Q_m for a single-cylinder research engine (black) and for cylinder 4 of a four-cylinder engine (gray).

tions and analyze relationships between successive cycles and [14, 15] quantify these with linear correlation coefficients. Linear controllers for reducing CV are developed in [17] using variable valve timings and in [18] using fuel injection timing. The control is based on model linearization which captures the trend that one cycle with early combustion phasing is followed by a cycle with late phasing and vice versa. In [19], the trends in [14–18] are confirmed but it is also shown that, for higher CV, the cycle evolution is more complicated and is characterized by different patterns than treated in these works. Specifically, the relationship between cycles exhibits strong nonlinear behavior mainly due to the sharp falloff in the combustion efficiency for late phasing. The analysis method from [19] is improved here and applied on data from a four-cylinder engine. The results are interpreted and compared to data from the single-cylinder experiments in [19]. As a preliminary comparison, Fig. 1 shows return maps for combustion phasing and heat release from one cylinder in the four-cylinder engine and from the single-cylinder. Return maps show the relationship between values of the variables in successive engine cycles and are a useful qualitative tool to study dynamic behavior. For example, return maps reveal nonlinear relationships that cannot be detected with linear correlation coefficients. Figure 1 shows that there are similarities in the dynamic behavior, especially for the phasing, despite the different engine platforms and operating conditions. Accordingly, the aim here is to quantify the coupling between engine cycles and the influence of variations between cylinders. To this end, the cycle evolution is investigated by analyzing thermodynamic variables and return maps. Symbolic-time series analysis is used to quantify the probabilities that certain sequences of cycles occur and enables a comparison between the four-cylinder engine and the single-cylinder. It is thereby demonstrated that when CV increases, the evolution is dominated by the low-order deterministic coupling that is due to interactions between the engine cycles rather than between the cylinders in HCCI engines with re-compression.

Coupling between cylinders in a four-cylinder HCCI engine controlled with exhaust throttles was analyzed in [20] through simulations and experiments. The coupling is found to be due to back-flow in the intake manifold causing an increased pressure in the intake manifold. In contrast to these results, in experiments in [15] for a six-cylinder engine operating with nvo no communication was detected between cylinders that affected combustion phasing. Contributing factors to the different conclusions could be the different engine setups and that the effect observed in [20] was strongly affected by the intake valve timings. Our analysis show that the coupling between cycles is similar between the four-cylinder engine and the single-cylinder while the large differences in the level of CV between cylinders are attributed to the high sensitivity at the edge of stable operation.

In the following two sections, the experimental conditions and the combustion analysis method are described. After that, the experimental results from the four-cylinder engine are interpreted and subsequently compared to data from a single-cylinder engine. The paper closes with the conclusions from the findings.

EXPERIMENTAL METHOD

Experiments were carried out in the Automotive Laboratory at the University of Michigan on a four-cylinder 2.0 L engine running on Tier-II certification fuel. The engine is a GM Ecotec modified for HCCI operation with increased compression ratio (to 11.25), modified camshaft (with lower lift and shorter duration), and a smaller turbocharger (a Borg-Warner KP31). The experiments were performed at constant speed of 2000 rpm and coolant temperature of 90°C. The combustion limits were explored by varying nvo and studying the influence of the nvo on the CV. Nominal load were 4 bar net indicated mean effective pressure (IMEP). Fuel injection started at 60° after top dead center nvo (aTDCn). The chosen conditions were highly diluted and fairly lean with approximately 50% residual gas and a relative air-fuel ratio λ of 1.3–1.4. The focus here is on the combustion behavior without spark, however, spark ignition must be enabled to avoid fouling and is therefore set to 25° after top dead center main combustion (aTDCm) to ensure minimal influence on the combustion. Also, the influence of the spark is small during such diluted conditions. Cylinder pressure was sampled ten times per crank angle degree for 3000 engine cycles. The large number of cycles is necessary for observing the cycle dynamics in detail as noted in studies of high CV in lean SI and in stoichiometric HCCI [6, 21].

COMBUSTION ANALYSIS

The analysis is based on standard methods for analysis of cylinder pressure combined with a method for estimating the unburned fuel on a per-cycle basis. This method was introduced in [19], and it is also shown here that the estimates converge independently of the guess for the unknown initial unburned mass. The estimation of the unburned fuel, which is the novel component in the combustion analysis, is described below whereas a detailed description of the complete estimation routine, including charge

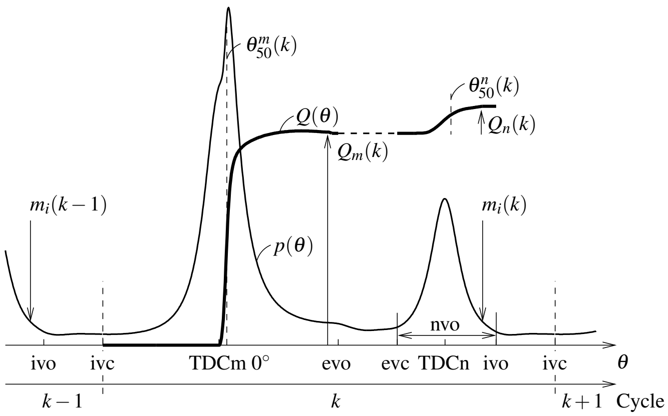

FIGURE 2. Definition of the engine cycle and important variables for curves of pressure $p(\theta)$ (thin line) and heat release $Q(\theta)$ (thick line).

mass estimation and heat release analysis, is found in [7, 19]. The most important assumptions are that the gas flow through the engine is constant and that the heat transfer is described by the modified Woschni correlation from [22].

The engine cycle and important variables are defined in Fig. 2 for curves of pressure $p(\theta)$ and accumulated heat release $Q(\theta)$. Note that the heat release analysis is applied on both the main combustion event, between intake valve closing (ivc) and exhaust valve opening (evo), and the nvo period, between exhaust valve closing (evc) and intake valve opening (ivo). The top dead center of the main combustion is defined as 0° and a cycle is defined to start at ivc. The total heat release in cycle k during main combustion and the nvo period are denoted by $Q_m(k)$ and $Q_n(k)$, and the 50% burn angles are denoted by $\theta_{50}^m(k)$ and $\theta_{50}^n(k)$, respectively.

The analysis routine is iterated until the combustion efficiencies, defined later in Eq. (8), converge at which point the governing relations of mass and energy conservation are satisfied. Although it is shown that the choice of initial values is not critical, a good initial guess, e.g. based on the average exhaust composition, may reduce the required number of iterations. Here, the values for complete combustion are employed. The main outputs from the routine are the thermodynamic state, pressure and temperature, and cycle-resolved values of unburned fuel and combustion efficiencies.

Unburned fuel

The unburned fuel mass in each engine cycle is estimated based on a difference equation for mass conservation that represents the evolution of the unburned mass. The inputs are injected fuel mass, heat release values, and residual gas fraction which are known from the other steps in the estimation routine [19]. The estimation requires an initial guess but it is shown below that the method converges independently of the value of the initial guess. The estimate is subsequently used to obtain estimates of combustion efficiencies on a per-cycle basis. The algorithm and its convergence are discussed below.

The fuel mass $m_f(k)$ in the beginning of cycle k (at ivc) is

$$m_f(k) = m_i(k-1) + m_u(k) \tag{1}$$

where $m_i(k-1)$ is the fuel mass injected in the nvo period of the previous cycle and $m_u(k)$ is the unburned fuel mass, in this cycle, carried over from previous cycles. The fraction $\eta_m(k)$ of $m_f(k)$ is consumed during the main combustion and, at evc, the fraction $x_r(k)$ of the residual gases is trapped. Of the remaining fuel mass, the fraction $\eta_n(k)$ is consumed during the nvo period. The unburned fuel mass carried over to the next cycle $k+1$ is then

$$m_u(k+1) = x_r(k)\,(1-\eta_m(k))\,(1-\eta_n(k))\,m_f(k) \tag{2}$$

based on the assumption that the residuals are homogeneously mixed. The combustion efficiencies in (2) are unknown but can be related to the total heat release. The gross heat released during the main combustion Q_m is calculated using cylinder pressure data [19] and is related to the fuel mass and efficiency η_m as

$$m_f(k)\eta_m(k) = \frac{Q_m(k)}{q_{lhv}} \tag{3}$$

where q_{lhv} is the lower heating value for the fuel. Similarly, the gross heat released during the nvo period Q_n is used to obtain:

$$m_f(k)\eta_n(k) = \frac{Q_n(k)}{(1-\eta_m(k))x_r(k)q_{lhv}} \tag{4}$$

Note that (3) and (4) are based on assuming that the energy release is proportional to the mass of fuel consumed and that any incomplete intermediate combustion reactions will be aliased into the mass estimates. Combining Eqs. (1)–(4) yields the final equation for predicting the unburned fuel mass:

$$m_u(k+1) = x_r(k)\left(m_i(k-1) + m_u(k) - \frac{Q_m(k)}{q_{lhv}}\right) - \frac{Q_n(k)}{q_{lhv}} \tag{5}$$

The inputs to this model are injected fuel m_i together with the total gross heat releases (Q_m, Q_n) and residual gas fraction x_r that are calculated according to [7, 19]. The fuel injection is constant here, $m_i(k) = \bar{m}_i$, since constant fueling was used in the experiments. Given these inputs, Eq. (5) is iterated given an initial condition $m_u(0)$. This value is unknown but, as shown below, it turns out to be of little significance since its influence quickly vanishes.

Convergence Equation (5) can be compactly written as

$$m_u(k+1) = x_r(k)m_u(k) + a(k) \tag{6}$$

where the coefficient $x_r(k)$ and the forcing term $a(k)$,

$$a(k) = x_r(k)\left(\bar{m}_i - \frac{Q_m(k)}{q_{lhv}}\right) - \frac{Q_n(k)}{q_{lhv}},$$

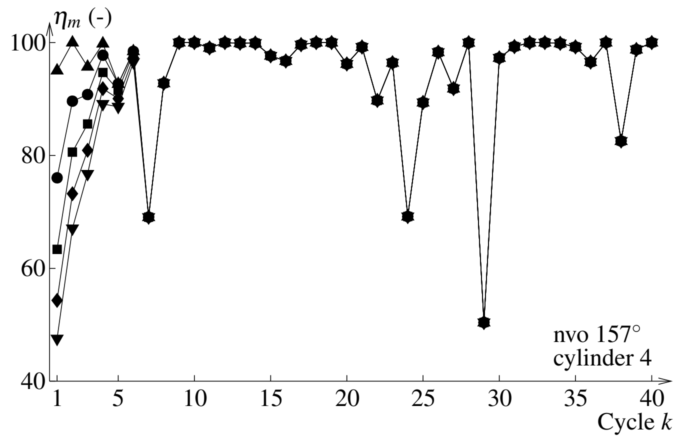

FIGURE 3. Estimates of the efficiency η_m for 40 cycles from cylinder 4. The symbols (▲,●,■,◆,▼) represents initial conditions $m_u(0)$ equal to (0,25,50,75,100)% of the injected fuel mass $\bar{m}_i$.

are known and vary with cycle. The general solution to the difference equation (6) for $k = 1, 2, \ldots$ is:

$$m_u(k) = a(k-1) + \sum_{i=0}^{k-2} a(i) \prod_{j=i+1}^{k-1} x_r(j) + m_u(0) \prod_{i=0}^{k-1} x_r(i) \qquad (7)$$

Since $0 < x_r(k) < 1$ holds in practice, the term with $m_u(0)$ eventually vanishes and it can be concluded that the series (5) is convergent. The unknown initial condition $m_u(0)$ can thus be guessed since, whatever the guess, its influence will vanish and the result will be unique after a transient. Moreover, it is seen from (7) that the convergence rate depends only on the value of $x_r(k)$. For the influence of the initial condition to be reduced to 10^{-m}, $n = \lceil -m \log_{10}^{-1} x_r \rceil$ iterations are required.

Combustion efficiencies When the unburned fuel mass is computed, (3) and (4) give

$$\eta_m(k) = \frac{Q_m(k)/q_{lhv}}{m_f(k)}, \quad \eta_n(k) = \frac{Q_n(k)/q_{lhv}}{x_r(k)\left(m_f(k) - \dfrac{Q_m(k)}{q_{lhv}}\right)} \qquad (8)$$

as approximations for the efficiencies. Due to the unknown initial condition $m_u(0)$, these are valid for $k \gtrapprox 12$. The influence of the unknown $m_u(0)$ is considered negligible when its influence on $m_u(k)$ is reduced by 1/1000. In this work, the value of x_r is not higher than 54% and, from the discussion in the previous paragraph, 12 cycles are thus sufficient. The convergence is illustrated in Fig. 3, on experimental data for a condition with high CV, for initial conditions $m_u(0)$ ranging from 0–100% of $\bar{m}_i$. Based on these results, an initial guess of $m_u(0) = 0$ is applied and the first 12 cycles (of the 3000 cycles of data) are discarded in the analysis.

nvo	160°				157°			
Cylinder	1	2	3	4	1	2	3	4
x_r (%)	55	55	53	53	53	53	50	51
θ_{50}^m (°)	7.6	8.6	7.1	9.0	11.0	12.1	11.8	12.5
$\sigma_{\theta_{50}^m}$ (°)	1.2	1.2	1.3	1.4	1.9	4.0	3.3	4.9
$\mathrm{CoV_{IMEP}}$ (%)	1.5	1.5	1.3	1.4	2.4	9.0	6.7	7.9

TABLE 1. The residual gas fraction (x_r), combustion phasing (θ_{50}^m), standard deviation of θ_{50}^m ($\sigma_{\theta_{50}^m}$), and coefficient of variation of IMEP ($\mathrm{CoV_{IMEP}}$) for two selected operating conditions.

RESULTS

This thermodynamic analysis is applied to several operating conditions at the boundary of stable combustion at late combustion phasing. Two representative operating conditions with different nvo are selected and reported here. The measured and estimated quantities are first examined revealing and quantifying the dominant coupling and cause for the behavior at conditions with high CV. Additionally, the cycle evolution is analyzed using return maps and symbol statistics. These mathematical tools provide a systematic method to discover dynamic interactions and reveal unstable behavior in phasing and heat release.

The first operating condition has an nvo of 160° and acceptable CV while the second condition has an nvo of 157° and high CV. The nominal experimental conditions are given above while Tab. 1 shows a number of variables that describe the change when reducing the nvo while keeping other actuators constant. With the smaller nvo the residual gas fraction decreases 2–3% for all cylinders and λ, measured with one sensor in the exhaust, increases from 1.3 to 1.4. The phasing θ_{50}^m retards 3.4–3.5°, except for cylinder 3 that retards 4.7° from TDCm. The standard deviation $\sigma_{\theta_{50}^m}$ and the coefficient of variation of IMEP $\mathrm{CoV_{IMEP}}$ increase notably. The largest increases are seen for cylinders 2 and 4.

Thermodynamic phenomena

From the measured cylinder pressure and estimated temperature for the two cases shown in Fig. 4 it is seen that reducing the nvo increases the CV for all cylinders. The increased variability can be seen in the spread of the observed peak pressures and pressure rise rates during main combustion. In the nvo period for the high CV case, the estimated temperatures are similar or higher than the case with low CV and sometimes reach values similar to those during main combustion. Furthermore, the highest peak cylinder pressures, during main combustion and the nvo period, are observed for the case with high CV. These observations suggest that there is unburned fuel that produces heat release during re-compression and also that unburned fuel may carry over and release heat in the next main combustion event. The level of CV

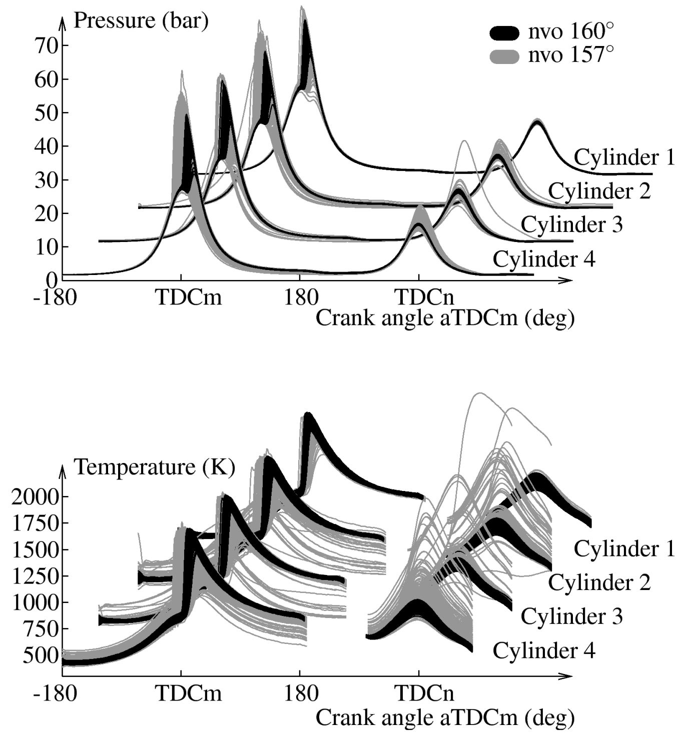

FIGURE 4. Pressure and temperature for low and high CV.

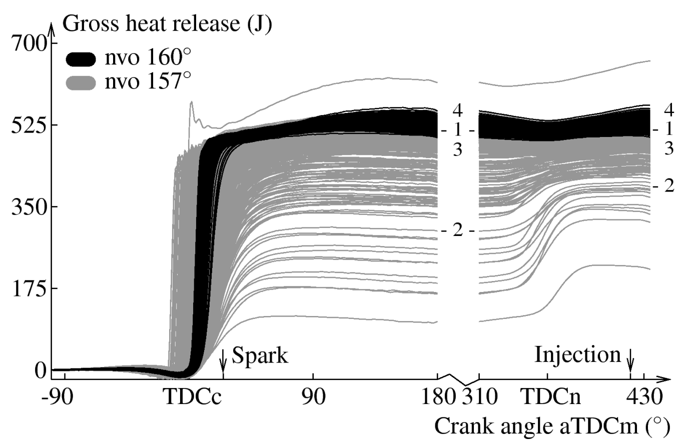

FIGURE 5. Gross heat release in cylinder 4 for low and high CV. Four consecutive cycles are numbered where all except the second cycle have similar total heat release. These cycles are also shown in Fig. 6–9.

differs between the cylinders, as seen in Fig. 4 and Tab. 1 where cylinder 1 has earlier phasing and much lower CV than the other three. To study the effects of CV in detail, the fourth cylinder, with highest $\sigma_{\theta_{50}^m}$, is studied in Fig. 5–7.

The gross heat release for cylinder 4 is shown in Fig. 5 for the two operating conditions. A coupling and complementarity of the heat release in the main combustion and the re-compression appears in our data. Cycles with a low accumulated heat release during main combustion are followed by a noticeable heat release during re-compression with a relatively slow burn rate. This is exemplified by the sequence of consecutive cycles marked in Fig. 5. The start of injection is late in the nvo period (60°aTDCn) and does not have any appreciable influence on the heat release, which justifies the use of Eq. (4). To clarify the cyclic coupling, Fig. 6 shows a number of variables versus the heat release during main combustion, $Q_m(k)$, in cycle k for cylinder 4 in the two operating conditions with different nvo. The top two rows show the main combustion phasing, $\theta_{50}^m(k)$, temperature at evo, $T_{evo}(k)$, heat release during the nvo period, $Q_n(k)$, and temperature at ivo, $T_{ivo}(k)$, versus $Q_m(k)$ in the same cycle k. The bottom row shows the phasing of the next cycle, $\theta_{50}^m(k+1)$, and the unburned fuel that is carried over to the next cycle, $m_u(k+1)$, versus $Q_m(k)$. When nvo decreases, CV increases, and trends appear of which the most evident are now summarized. Late phasing $\theta_{50}^m(k)$ in cycle k is associated with a low heat release $Q_m(k)$ which in turn leads to a decreased $T_{evo}(k)$. This decrease is, however, canceled by the nvo heat release so that $T_{ivo}(k)$ increases which advances the phasing $\theta_{50}^m(k+1)$ of the next cycle, $k+1$. Also, even if there is heat release during nvo, some of the unburned fuel, up to 8% as estimated by Eq. (5), carries over to the next cycle. The complementarity of the heat release in the main combustion and the nvo period has been observed earlier in experiments [14, 16] and predicted in simulations [23]. However, the estimates of the unburned fuel mass suggest that not all of the remaining fuel is necessarily consumed during nvo and a fraction of it may carry over to the next cycle. An example of these trends is the sequence of four cycles marked in Fig. 5 and 6.

Figure 7 shows the main combustion efficiency η_m and the ringing intensity I versus the main combustion phasing θ_{50}^m. The efficiency η_m is computed from Eq. (8) and the intensity I is computed according to the correlation developed by J. Eng [24, Eq. (16)]. The average I for all cycles is 4.5 $\mathrm{MW/m^2}$ and 2.8 $\mathrm{MW/m^2}$ for the low and high CV case, respectively. Although the average is lower, the spread of I in an individual cycle is much larger than that for the low CV. The efficiency η_m decreases with later phasing whereas the intensity I increases with early phasing, as expected. These early cycles have high I due to large pressure oscillations that may increase the heat transfer. This effect is not captured in the heat transfer model, which is the modified Woschni from [22]. An underestimated heat transfer will lead to a lower estimated value of η_m, see Eq. (8), and this is suspected to be a contributing factor to the drop in η_m for early phasing. The sharp drop in efficiency at very late phasing shows that the sensitivity with respect to phasing increases dramatically. Thus, the efficiency curve, which determines the unburned fuel mass after main combustion, is an important nonlinear relation for accurately describing the coupling between cycles. The importance is further illustrated in the following analysis using return maps.

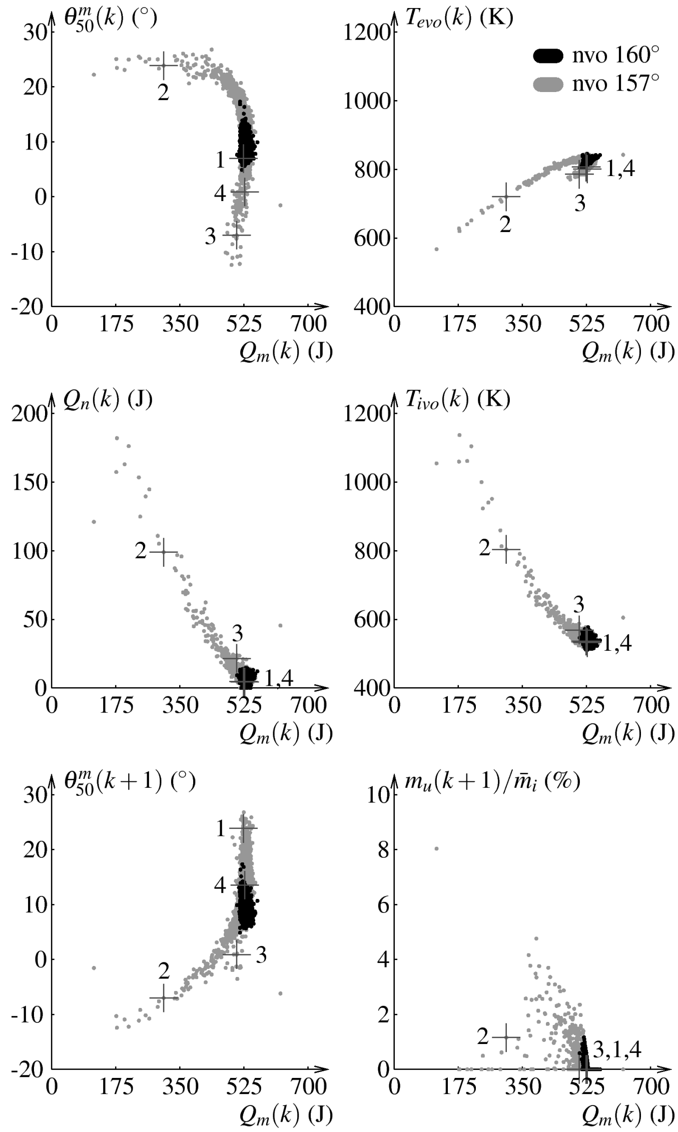

FIGURE 6. Cylinder 4 heat release during the main combustion, $Q_m(k)$, versus main combustion phasing, $\theta_{50}^m(k)$, temperature at evo, $T_{evo}(k)$, heat release during the nvo period, $Q_n(k)$, temperature at ivo, $T_{ivo}(k)$, combustion phasing, $\theta_{50}^m(k+1)$, and unburned fuel $m_u(k+1)$.

Return maps

The causality of important variables and the influence of Q_m were shown in Fig. 6. Another useful graphical tool for studying the cycle evolution of variables are return maps, which show the relationship between subsequent values of a variable. Most importantly, return maps reveal low-order deterministic features even when these are nonlinear.

A return map or lag-1 map, shown in Fig. 8 and 9, is a plot of a variable $x(k)$ at cycle k versus the value at the next cycle $x(k+1)$. Figure 8 shows the return maps for combustion phasing

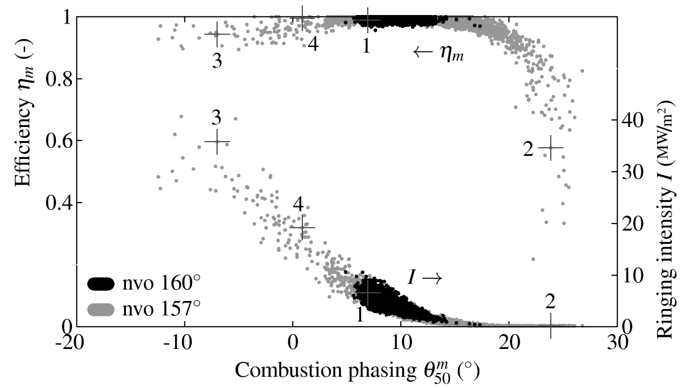

FIGURE 7. Combustion efficiency, η_m, and ringing intensity, I, versus phasing for low and high CV showing the increased spread of values for high CV. The numbered cycles are also shown in Fig. 5, 6, 8, 9.

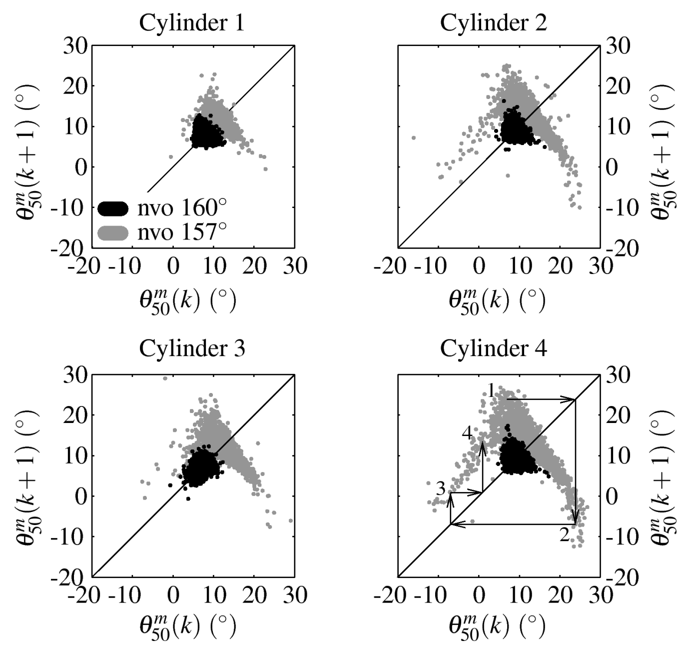

FIGURE 8. Return maps for phasing θ_{50}^m. The marked sequence for cylinder 4 corresponds to the numbered cycles in Fig. 5–7, 9.

θ_{50}^m for the two operating conditions studied. For nvo=160°, the return maps show clouds without clear structure. With nvo=157°, the CV increases and characteristic patterns emerge. The patterns are more pronounced for the cylinders with higher CV but all share the same features. The main characteristics of the patterns can be described by a cluster of cycles stretching out perpendicular to the diagonal and another part bending the left tip and extending down to the left. The perpendicular part indicates an unstable behavior where θ_{50}^m alternates between early and late phasing. The oscillations in θ_{50}^m can have a large amplitude as shown by the magnitude of the perpendicular cluster of cycles

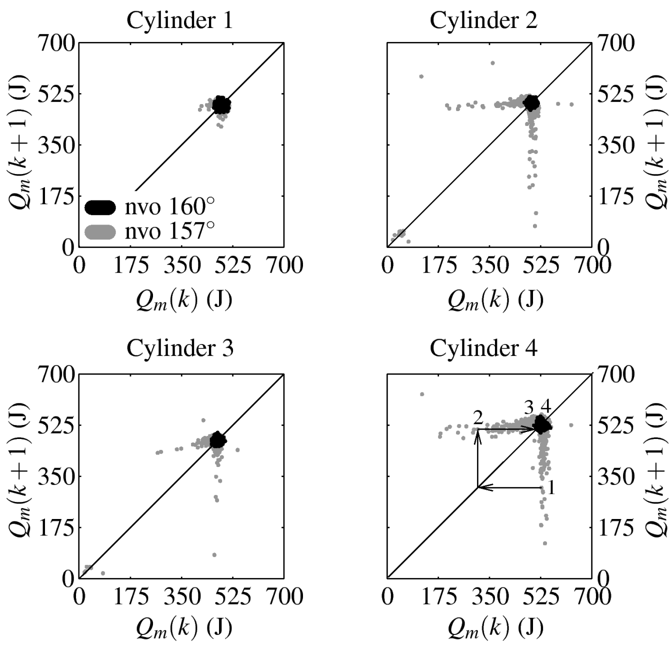

FIGURE 9. Return maps for total heat release Q_m. The marked sequence for cylinder 4 corresponds to the numbered cycles in Fig. 5–8.

which is most pronounced in cylinder 4. The right part of the cluster together with the bent part show that there is a limit for the amplitude value so that when the amplitude is sufficiently large the combustion does, at least temporarily, recover to a phasing closer to the average. The sequence discussed earlier for cylinder 4 in the high CV case, is marked in Fig. 8 and is an illustration of a large amplitude oscillation followed by a recovery. The overall patterns for θ_{50}^m for high CV show a clear asymmetry about the diagonal. An asymmetric pattern implies an asymmetry in time which means that the evolution of θ_{50}^m is due to nonlinear dynamics or non-Gaussian noise, or both [25].

The patterns for the heat release Q_m, shown in Fig. 9, are characterized by two parts, one stretching out vertically and another extending to the left. An unstable behavior is also seen here in the vertical part indicating that a cycle with an average heat release can be followed by a cycle with a large range of values. This is related to the fact that the efficiency and, thus, Q_m have a very steep falloff for late phasing, as demonstrated in Fig. 6–7. A late cycle with low heat release is, in turn, followed by a cycle with approximately average heat release as indicated by the left part of the pattern that stretches out almost horizontally. The sequence marked for cylinder 4 in Fig. 7–9 illustrates these trends.

The characteristics for the evolution of the combustion phasing show that large excursions are preceded by an unstable oscillation with large amplitudes. On the other hand, the heat release showed an unstable behavior where there could be a large change from one cycle to the next which is a result of the nonlinear efficiency curve, see Fig. 7 where it is shown that the phasing may

Symbol	Definition
(0) very early	$-12 < \theta_{50}^m < 10$
(1) early	$10 < \theta_{50}^m < 12$
(2) median	$12 < \theta_{50}^m < 14$
(3) late	$14 < \theta_{50}^m < 16$
(4) very late	$16 < \theta_{50}^m < 27$

TABLE 2. The partitioning of the combustion phasing data, from cylinder 4 in the case with high CV, such that each symbol is equally probable.

vary in a fairly large range with similar efficiency but it drops off sharply when the phasing passes a threshold.

Symbol statistics

Symbolic time-series analysis is a technique for analyzing noisy data especially for nonlinear and possibly chaotic systems, see [26] for a presentation of the technique for studying combustion data. The return maps give important insights into the dynamic behavior between successive cycles and primarily in a qualitative manner while symbolic time-series analysis is a method to study sequences of several cycles and also to quantify the probability for certain sequences to occur. This quantification enables the comparison of the most probable sequences observed among different cylinders and engines. In particular, the symbol statistics allow us to compare the high CV behavior at late-phasing in the multi-cylinder engine with the single-cylinder.

To obtain the symbol statistics, the measurement values, e.g. θ_{50}^m, are first partitioned into a set of n discrete symbols and then the statistics for the occurrence of symbol sequences $s = s_1 s_2 \dots s_L$ of length L are computed. The partitioning is done so that every symbol is equally probable. The relative frequency for any order of cycles is then n^{-L} if the evolution is purely random. In other words, the values of combustion phasing θ_{50}^m and heat release Q_m are categorized into n bins and the boundaries of the bins are chosen so that the number of samples are the same in each bin.

The number of categories, the symbol-set size n, is chosen so as to discriminate data points that have extreme values from the ones close to the median or the ones with intermediate values. The symbol-set size n is chosen to be 5, which gives reasonable granularity for phasing and heat release. Consequently, this choice distinguishes between very early, early, median, late, and very late values of the combustion phasing. For the heat release, the categories are very low, low, median, high, and very high values. For a compact representation, these categories are labeled with a number from 0 to 4. As an example, the partitioning for cylinder 4 in the high CV case is shown in Tab. 2. The intervals for the very early and very late categories are wider than the others due to the equiprobable partitioning and the fact that there are more

Sequence	Description
$s = 140$	An early cycle (1) is followed by a very late cycle (4) which is followed by a very early cycle (0).
$s = 403$	A very late cycle (4) is followed by a very early cycle (0) which is followed by a late cycle (3).

TABLE 3. Examples of symbol sequences.

data points close to the median than farther away. Note that the partitioning is different for each data set meaning that each cylinder in the respective case has its own partitioning due to the equiprobable requirement.

The symbol-sequence length L is chosen to be 3 by using the approach in [26] utilizing a modified Shannon entropy. Roughly speaking, the aim is to choose L so as to best identify the deterministic features in the data. Using the labeling of the categories with a number from 0 to $n-1=4$, the cycle evolution is compactly described by sequences of $L=3$ of these numbers. To illustrate, consider the sequences (140,403) for the phasing θ_{50}^m. These represent sequences of cycles $s = s_1 s_2 s_3$ where the values of θ_{50}^m are categorized as shown in Tab. 3.

The symbol statistics for the combustion phasing θ_{50}^m and for the heat release Q_m are shown in Fig. 10–11 where the four sequences with highest probability are marked in the figure. Note that a sequence is a number in the base $n=5$ while the abscissa in Fig. 10–11 is, for convenience, in base 10. For example the sequence 140 in base 5 is 45 in base 10.

Figure 10 shows that, when reducing the nvo, sequences appear that have probabilities clearly higher than $n^{-L} = 0.8\%$ (shown with the dashed line) for a random evolution and this is most evident for cylinders 2 and 4 that have the highest CV, see Tab. 1. The two most common sequences of θ_{50}^m for all cylinders that emerge at high CV are 040 and 404, representing oscillations between the extreme values of the partitioning. The sequence 140 is the third most common and is a sequence with increasing amplitude, see Tab. 3. These sequences correspond to the qualitative features discussed above for the return maps of unstable oscillations. Note that it also can be concluded that common sequences are combinations such as 0404, 4040, and 1404. For example, after 040 one of the sequences (400,401,402,403,404) must follow of which 404 has the highest probability. The fourth most common sequence is 403 for cylinders 1 through 3 and common combinations are thus also 0403 and 1403. In contrast to 140, 403 is a sequence with decreasing amplitude, see Tab. 3. For cylinder 4, with the highest CV, the probability of 403 sequences are diminished and 131, an oscillation where the amplitude is approximately sustained, is instead the fourth most common sequences of cycles. Note that for cylinder 2 and 4 the sequences that are more pronounced for

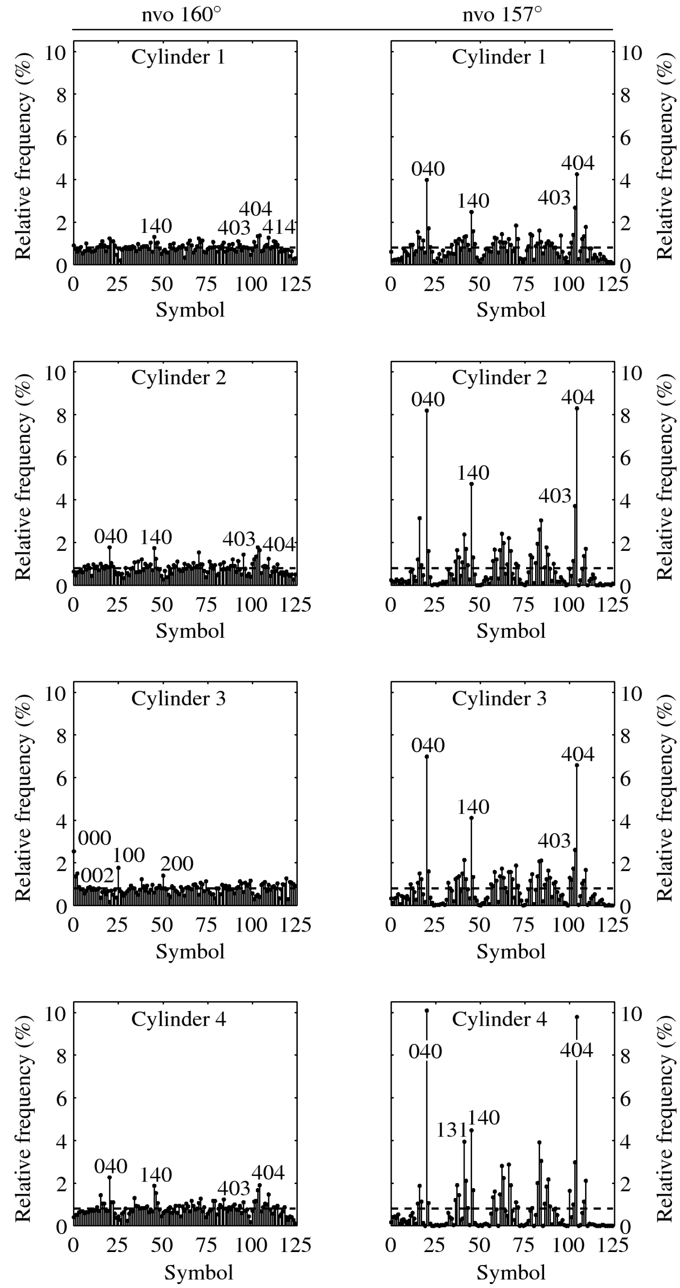

FIGURE 10. Symbol statistics for combustion phasing θ_{50}^m.

higher CV, i.e. with nvo of 157°, already can be distinguished for lower CV, i.e. with nvo of 160°. This cannot be easily determined by looking at the return maps in Fig. 8 or the measures $\sigma_{\theta_{50}^m}$ and $\mathrm{COV_{IMEP}}$ in Tab. 1. The asymmetry found in the return maps can also be seen in the symbol statistics by frequency mismatches between time inverses of sequences [26], here exhibited by the sequences 140 and 403.

The statistics for the gross heat release Q_m are shown in

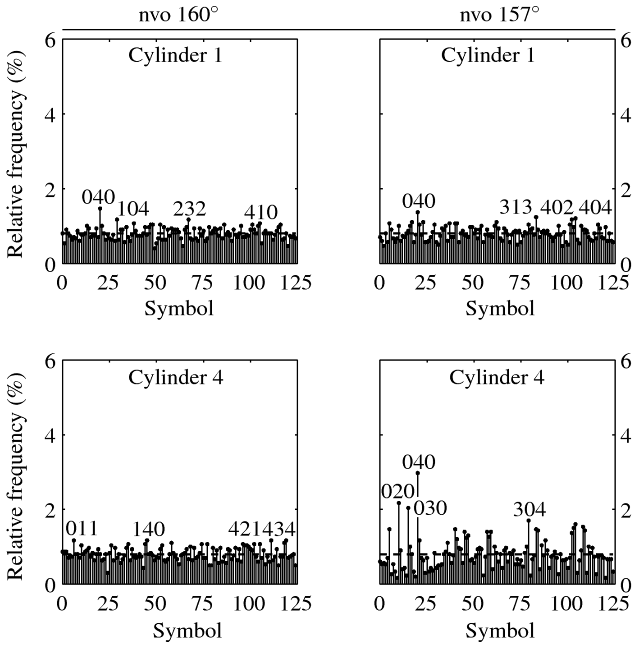

FIGURE 11. Symbol statistics for the gross heat release Q_m.

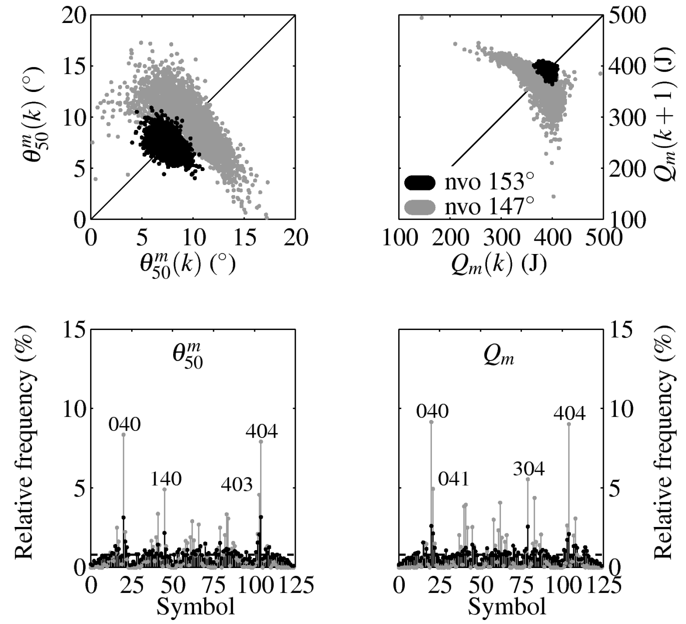

FIGURE 12. Return maps and symbol statistics for single-cylinder experiments reported in [19].

Fig. 11, cylinder 1 and 4 are shown to conserve space. Distinctive sequences appear in cylinder 4 where oscillating sequences emerge with approximately sustained amplitude (020,040,030) and with increasing amplitude (304). The most common sequences for cylinder 2 and 3, which are not shown, are (030,040,304,404) and (020,040,041,404), respectively, which show the same characteristics with the exception of the sequence with reduced amplitude (041) for cylinder 3.

COMPARISON WITH SINGLE-CYLINDER DATA

The cycle evolution is here investigated in experimental data from [19] from a single-cylinder research engine and compared with the multi-cylinder data qualitatively using return maps and quantitatively using symbol statistics. The single-cylinder experiments were performed on an engine with a compression ratio of 12.4 running on research grade gasoline. The nominal operating conditions were λ = 1.7, net IMEP of 2.8 bar, and 2000 rpm. Despite the different engine platforms and conditions, the dynamic behavior for high CV shows strong similarities suggesting that the governing coupling is between cycles of the individual cylinders.

It should be noted that the estimated combustion phasing is not as sensitive as the value of the gross heat release to the errors in the estimated heat transfer that were discussed in connection with the efficiency estimates. Also, the estimated phasing is robust against noise due to the fast nature of HCCI combustion [27]. For these reasons, stronger emphasis is put on the comparison of the combustion phasing evolution than the heat release evolution.

Figure 12 shows the return maps and symbol statistics for phasing θ_{50}^m and heat release Q_m for two settings of nvo transitioning from low to high CV. The symbol-set size n is 5 and sequence length L is 3 as used above. Comparison of the return maps in Fig. 12 with Fig. 8 and 9 shows that there are clear geometric similarities. In particular, the θ_{50}^m patterns are very similar and are described by one part perpendicular to the diagonal, indicating unstable oscillations, and a bent tip, showing a recovery behavior. This is confirmed and quantified by the symbol statistics that show similar frequency distribution and also good agreement for the dominating sequences. The return maps for the heat release show an unstable vertical direction in both cases but the part extending to the left shows different behavior. For the four-cylinder case, in Fig. 9, a cycle with low heat release, far down the vertical part, is followed by a heat release with a value close to but lower than the average. The single-cylinder data shown in the top right plot in Fig. 12, conversely to the four-cylinder engine data, shows that a cycle with low heat release is followed by a heat release higher than average. Comparing the symbol statistics for the heat release from the single-cylinder and from the four-cylinder engine shows that the agreement is not as good as for the phasing.

CONCLUSIONS

High CV conditions in a four-cylinder engine operating HCCI by negative valve overlap is analyzed and compared to single-cylinder data. The combustion analysis is applied on main combustion but also on the re-compression period, which is important for the understanding of conditions with high CV. The observations confirm previous conclusions from the single-cylinder experiments that the dominating effects are recycled thermal energy and recycled unburned fuel in the residual gases. Identifying these mechanisms are vital for model-based control approaches and the return maps show qualitatively that as the combustion phasing retards and the CV increases, the cycle evolution is governed by these low-order deterministic mechanisms rather than random or high-order dynamic behavior. Using symbolic time-series analysis, the findings are quantified in terms of probabilities for the most common sequences of cycles. It was also observed that symbol statistics can detect conditions with low CV, which cannot be observed with simpler variability measures, such as CoV of IMEP, or by looking at the return maps.

The qualitative behavior of the combustion phasing at high CV in each cylinder is found to be similar to the behavior observed in single-cylinder experiments. Due to the high sensitivity of the conditions at the edge of steady operation, small variations between cylinders are amplified at high CV conditions, and the variations between cylinders are thus expected to be higher than at low CV. Moreover, the results suggest that the in-cylinder dynamics are dominated by the coupling between cycles while the variations between cylinders, for example due to flow phenomena and different wall temperatures, strongly influence the level of CV. For feedback control, each cylinder could thus be viewed as a separate control loop and, moreover, the low-order models developed in [13] that predicted the behavior at high CV for a single-cylinder engine can capture the behavior in a multi-cylinder engine with the proper model inputs for each cylinder. The different gross heat release patterns in the single- and multi-cylinder engines remain to be explained. For control purposes, however, the similarity of the dynamic behavior of the combustion phasing, which can be robustly computed on-line with fairly low complexity from cylinder pressure data and used as a feedback signal, is very encouraging and the most important finding of this work.

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