

# Chapter 6

## Arrow/Vector Plots

### 6.1 Quiver/Arrow/Vector Plots/Porcupine Plot

To plot vectors, it is useful to draw arrows such that the direction in which the arrow points is the direction of the vector and the length of the arrow is its magnitude. Such plots are very popular in fluid mechanics, for example, where the flow velocity in a two-dimensional plane is often illustrated this way. This strategy can be applied in either two or three space dimensions, but is rarer in three dimensions because software is not as readily available and because the arrows in the foreground tend to obscure arrows farther from the viewer. Unfortunately, there is no standard name for this type of plot: “arrow”, “quiver” and “vector” are all in common usage, and “porcupine” is used occasionally. Fig. 6.1 is an example.

Advantages:

1. Easy, intuitive representation of vectors.

Disadvantages:

1. Small magnitude vectors are depicted as dots. At these points, direction information is lost.
2. High velocity vectors may overlap adjacent gridpoints so that the arrows dissolve into a sea of confusing line segments
3. Because of the difficulties with both large and small magnitude vectors, it is difficult to represent a wide range of magnitudes in an arrow plot.
4. When superimposed on pseudocolor plots, a common option, the vectors may be hard to see against the bright colors.

### 6.2 Vector Plots Superimposed on Other Graphs

It is very common to superimpose vector plots on other graphs such as quiver plots. In black-and-white, the most common “combo” is vector-plot-on-contour-plot as exemplified by Fig. 6.2. In fluid mechanics, the contours are a scalar quantity such as pressure or temperature; the vectors are horizontal velocity. When color is available, the contour plot is often replaced by pseudocolor graph or by a filled contour plot, which is the superposition of a contour plot and a pseudocolor graph of the same variable as in Fig. 6.3.

But what if one wants two scalar variables plus a vector? A good triple combo is to combine a pseudocolor plot of one with a contour plot of the second scalar plus a vector plot.

The great pitfall of “combo” plots is that the pseudocolor graph tends to visually overwhelm the vector field. Only a well-motivated reader who studies the graph for a few seconds will identify the patterns in the vectors. Furthermore, because the colors of the pseudocolor image are varying, some vectors which are superimposed on dull hues may be rather difficult to see.

Indeed, it was necessary to reduce the number of points in the grid to  $21 \times 21$  so that the vectors would be readily visible in Fig. 6.3. The only vector arrows which are easily seen against the background of bold reds, yellows, and blues are BIG arrows.

In black-and-white graphs, it is difficult to use the convention of showing negative-valued contours as dashed lines because the dashes are too faint to be easily distinguished from the vectors.

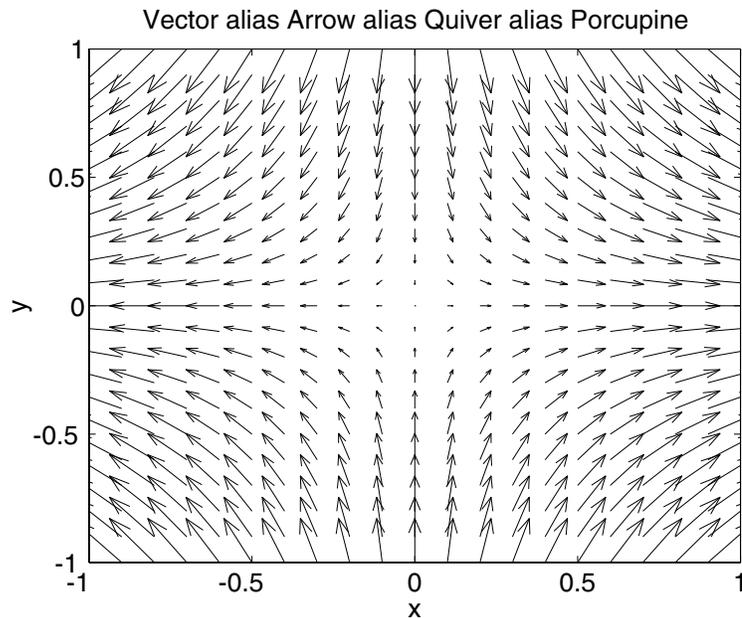


Figure 6.1: A schematic two-dimensional vector field, plotting by drawing arrows whose direction is that of that of the vector and whose length is proportional to the magnitude of the vector at that point in the  $(x, y)$  plane. The center of the field illustrates a generic defect of arrow plots: where the magnitude of the vector is SMALL, the arrows are so short that it is difficult to perceive either their direction or relative magnitude.

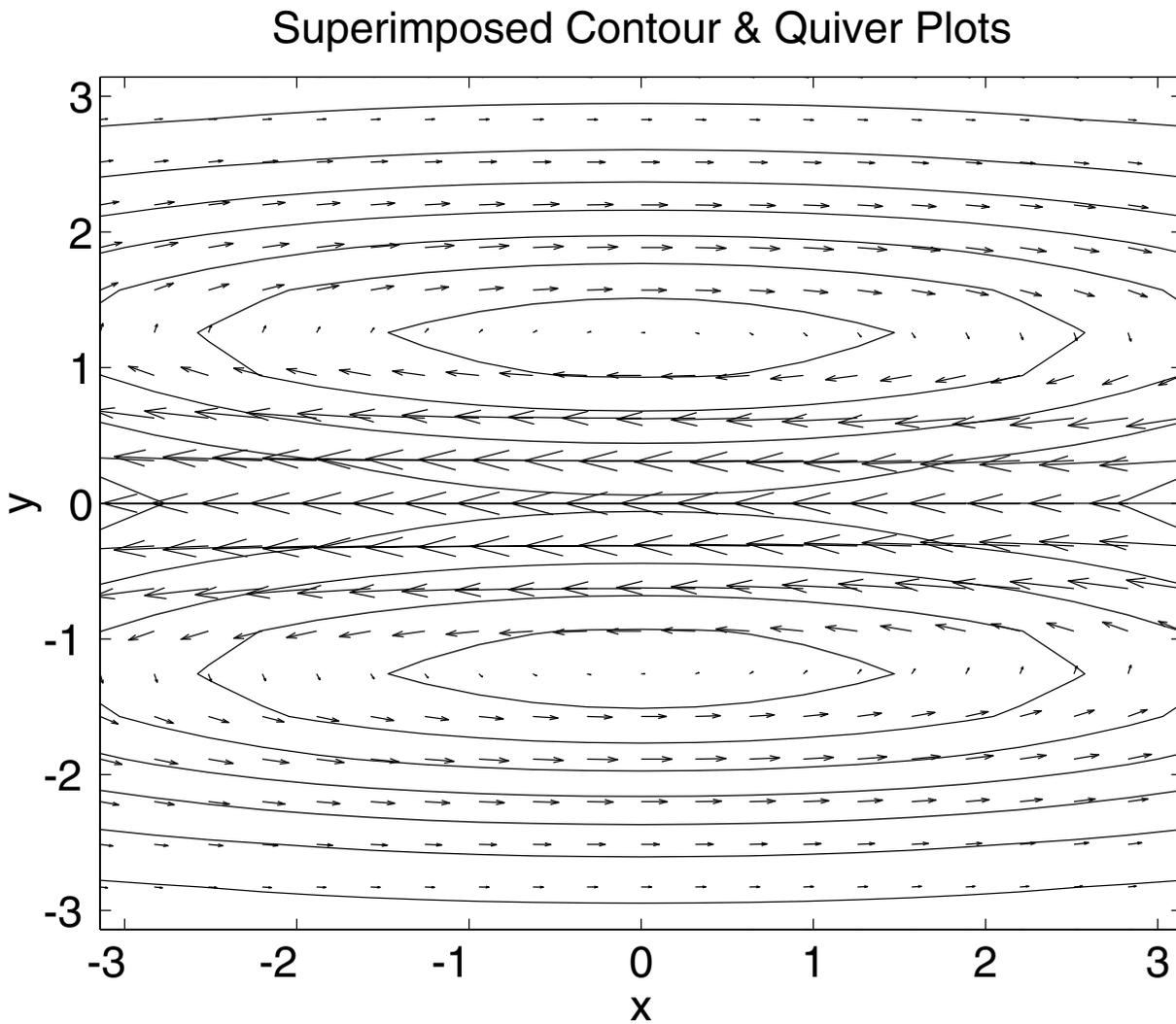


Figure 6.2: Quiver (vector) plot superimposed on a contour plot of a different field. The vectors show the horizontal velocity with the length of the vector equal to the magnitude of the vector. The contours of the pressure or height field. (Equatorial Rossby wave, plotted versus longitude ( $x$ ) and latitude ( $y$ )).

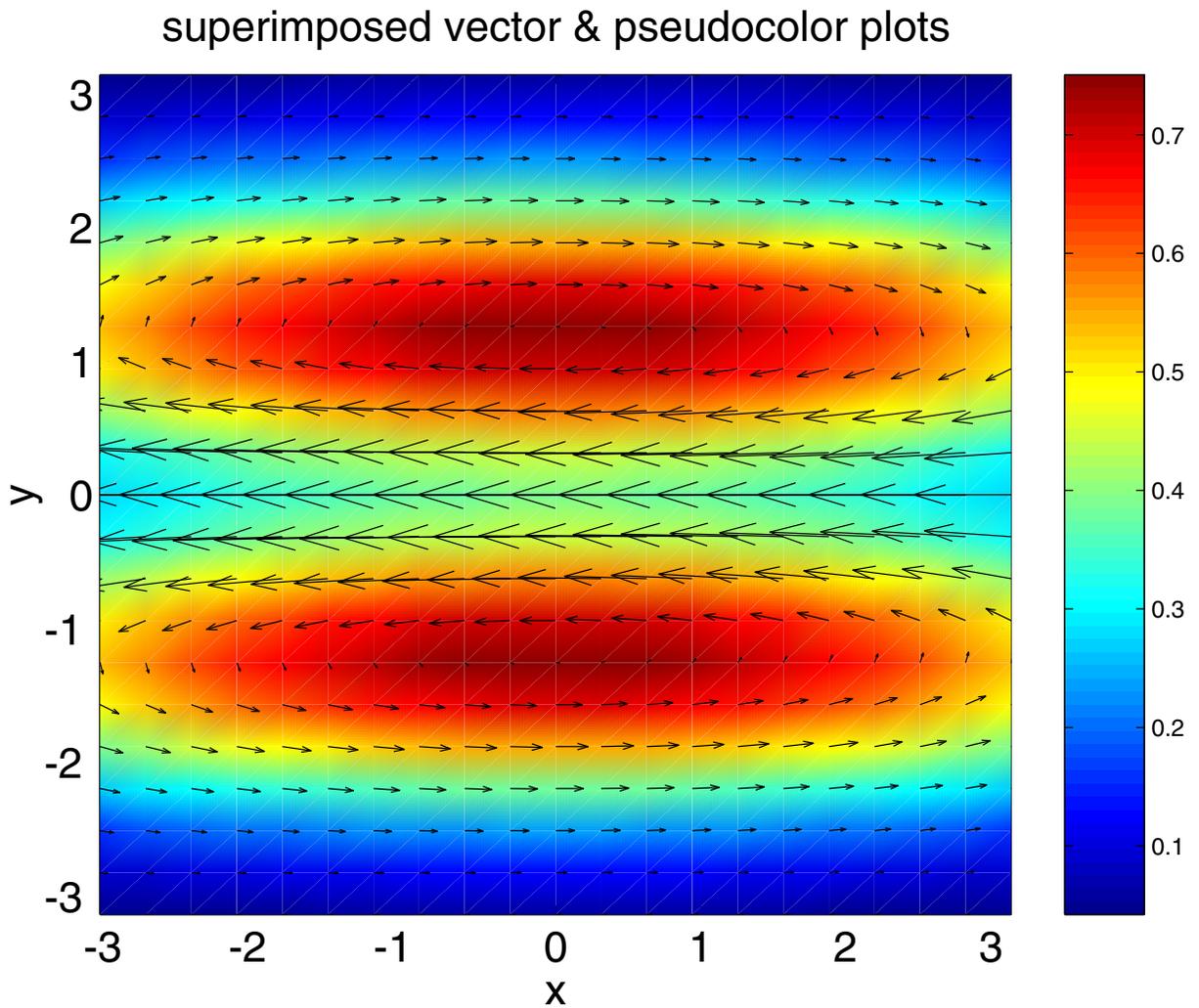


Figure 6.3: Same as the previous figure except that the contour plot is replaced by a pseudocolor graph of the pressure. The density of the grid for the vectors had to be reduced only  $21 \times 21$  so that the vectors would be sufficiently large to be visible when superimposed on the pseudocolor graph.

### 6.3 Vectors Derived from Scalars: Gradients and Hamiltonian-Field/Streamfunction Flow

Arrow plots are used not only to plot quantities that are vectors in and of themselves, but these species of graph can be applied to analyze SCALARS by plotting vector fields DERIVED from the scalar-valued quantity. In electrostatics, for example, the gradient of the potential is the electric field; in fluid mechanics, the gradient of the velocity potential gives the irrotational flow associated with the potential. The components of the gradient are

$$\left(\frac{\partial f}{\partial x}, \frac{\partial f}{\partial y}\right) \quad (6.1)$$

Similarly, in physics, one can define a “Hamiltonian gradient” which is a sort of skewed gradient. In fluids, the components of a nondivergent velocity field in two dimensions are related to the streamfunction  $\psi$ , which is a scalar, by

$$u = -\frac{\partial \psi}{\partial y}, \quad v = \frac{\partial \psi}{\partial x} \quad (6.2)$$

The derived vector fields are sufficiently important that Mathematica has one-line plot commands to generate an arrow plot of the derived vector from an input of an array of values of the scalar: **PlotGradientField** and **PlotHamiltonianField**. Matlab does not, but does contain a built-in function **[fx,fy]=gradient(f)**, which will compute the gradient through finite differences. This makes it possible to compute either the gradient or “Hamiltonian gradient” in two lines.

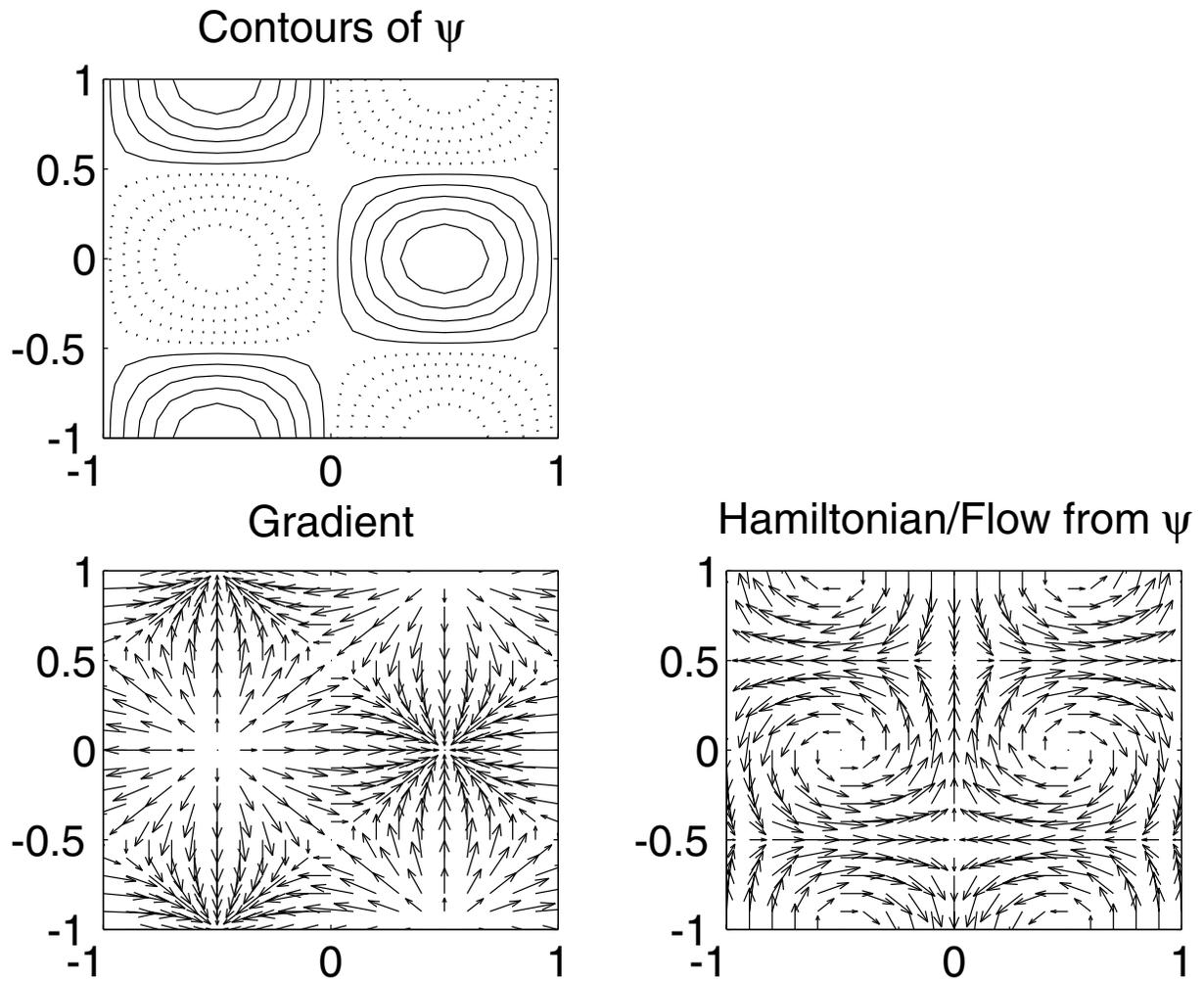


Figure 6.4: Upper left: contours of  $\psi = \sin(\pi x) \cos(\pi y)$ . Lower left: vector plot of the gradient of  $\psi$ ,  $(\psi_x, \psi_y)$ . Lower right: the vector field  $(-\psi_y, \psi_x)$ . This is called a “Hamiltonian gradient” by physicists and a “two-dimensional incompressible flow derived from the streamfunction  $\psi$ ” by fluid dynamicists.

## 6.4 Area Quiver Plots

In this variant, the *thickness* and *length* of the arrows are *both* varied so that the magnitude of the vector is proportional to the *area* of the arrow rather than its length. A schematic example is shown in Fig. 6.5.

Advantages:

1. One can represent a wider range of magnitudes without having the arrows shrink to dots or overlap one another.
2. strong currents are marked by thick arrows versus the thin arrows seen elsewhere, which makes the jets easier to identify.

Disadvantages:

Areas are more difficult for the eye to judge than lengths.

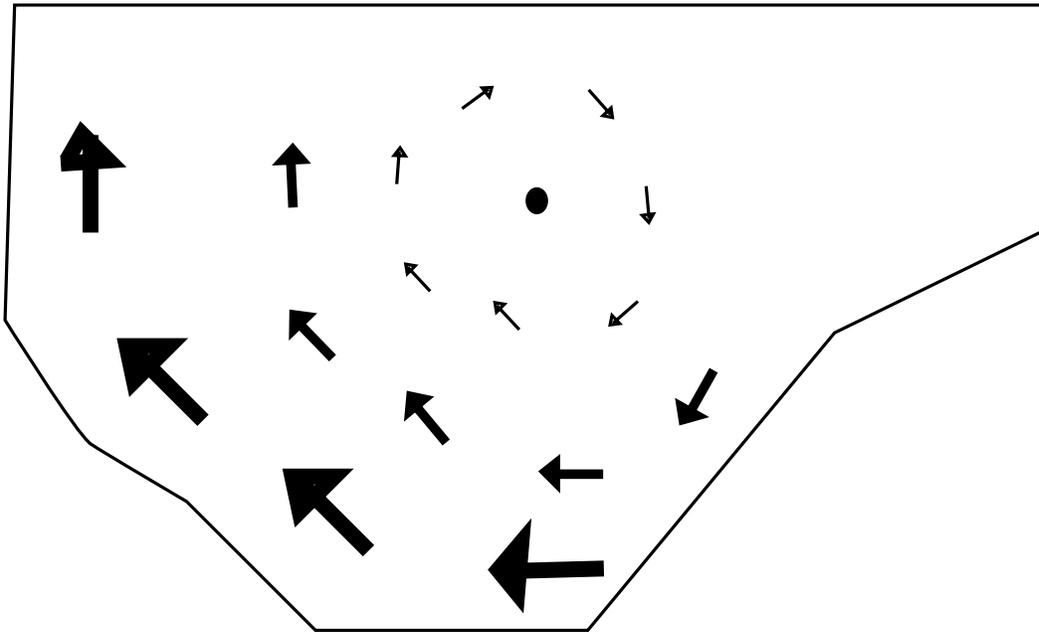


Figure 6.5: A variant of the vector or quiver plot in which the AREA of the vectors denotes the magnitude of the vector. (Note that this variant is rare; it is much more common to denote magnitude by the LENGTH of the vectors only.)

## 6.5 Three-Dimensional Arrow/Vector/Quiver Plots

Matlab has command **quiver3** to compute three-dimensional vector plots. Its syntax is

$$\text{linehandles}=\text{quiver3}(x,y,z,u,v,w);$$

where  $x, y, z$  are vectors of points that specify a point in space as  $(x(i), y(i), z(i))$  and where  $u, v, w$  are arrays that specify the components of the three-dimensional vector field at these points. (Because early versions of Matlab could not handle three-dimensional vectors, the **quiver3** function assumes that all its arguments are vectors; if the points lie on a rectangular array with  $nx$  points in  $x$ ,  $ny$  points in  $y$ , and  $nz$  points in  $z$ , then each of the six input vectors is of length  $nxnynz$ .) The function returns an array of linehandles which can be used to manipulate the plot.

The great disadvantage of three-dimensional quiver plots is that the projection of the three-dimensional array of arrows onto a two-dimensional flat screen or paper gives a very busy graph with limited depth cues so that is often impossible for the eye to make sense of it. Fig. 6.6, which shows the initial velocity for a celebrated fluid mechanics flow called the Taylor-Green flow, is an illustration. The Taylor-Green flow is an array of vortices, but one can derive only the vaguest sense of the vortices from the plot.

Another drawback is to have any hope of making sense of the vectors, the grid of arrows must be rather coarse. The Taylor-Green plot on a denser grid is even more incompressible, but the  $11 \times 11 \times 5$  grid illustrated is very coarse, and obviously makes many of the finer features of the vector field invisible even in principle. (In practice, it is often difficult to perceive even the coarsest features of the field.)

Because of these disadvantages, it is likely that widespread use of three-dimensional arrow plots will have to wait until either three-dimensional holographic computer displays or stereo computer monitor glasses are available.

Still, one can fiddle with three-dimensional quiver plots to improve visualizability. Fig. ?? shows the same plot as in the previous figure except that changes have been made to make the diagram easier to comprehend:

1. Grid is reduced to  $6 \times 6 \times 3$ .
2. Different vertical levels are color-coded: red at the top, black in the middle, and blue at  $z = 0$ .
3. Only  $1/8$  of the period box is illustrated.

The reduction of the volume which is illustrated by a factor of two in each dimension is possible because the Taylor-Green flow is the product of trigonometric functions of period two. It follows that knowledge of the flow in the box  $[0, 1] \times [0, 1] \times [0, 1]$  is sufficient to depict the flow in the remaining seven cubes of the period box.

## Initial velocity: Taylor-Green vortex

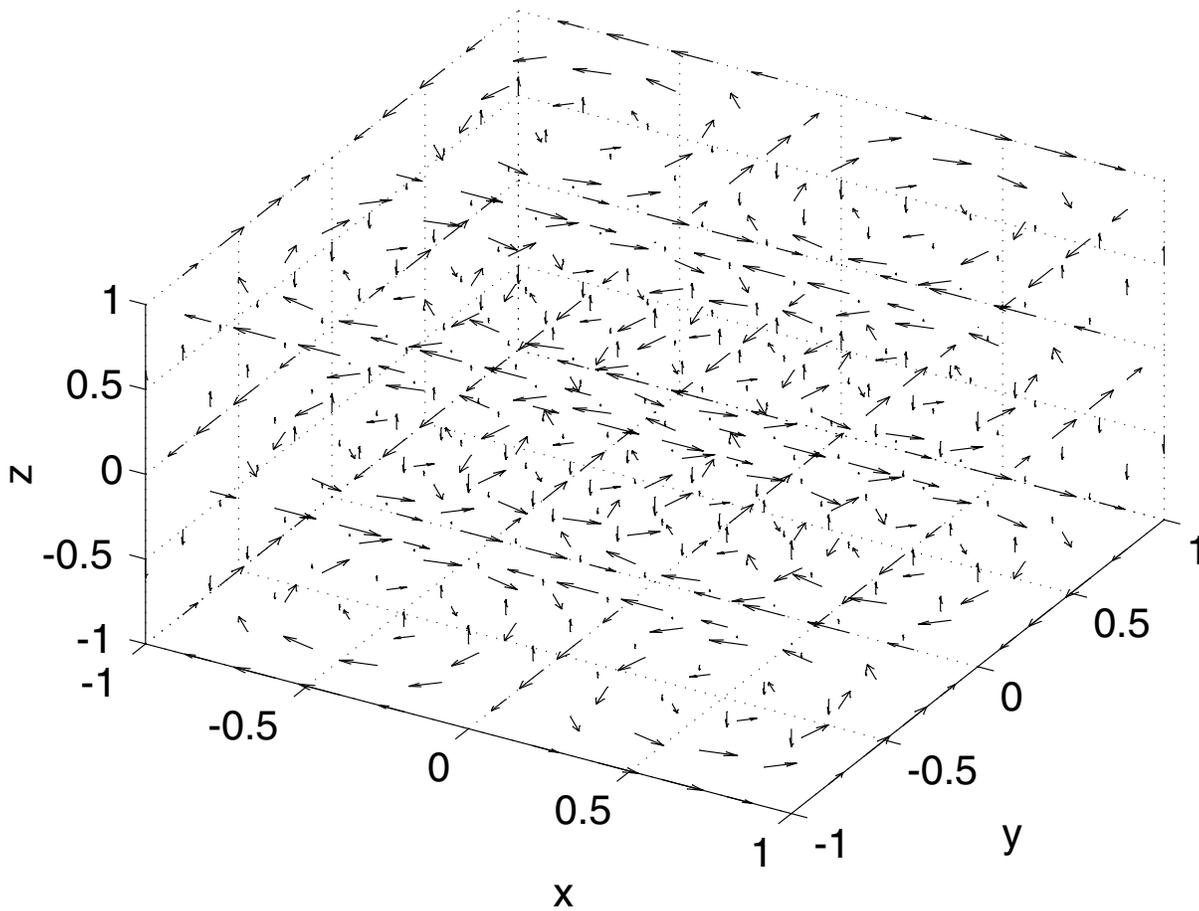


Figure 6.6: A three-dimensional vector field, the initial velocity for the Taylor-Green flow of incompressible, homogeneous turbulence theory. The components are  $u = \sin(\pi x) \cos(\pi y) \cos(\pi z)$ ,  $v = -\cos(\pi x) \sin(\pi y) \cos(\pi z)$ ,  $w = \sqrt{\frac{2}{3}} \cos(\pi x) \cos(\pi y) \sin(\pi z)$ .

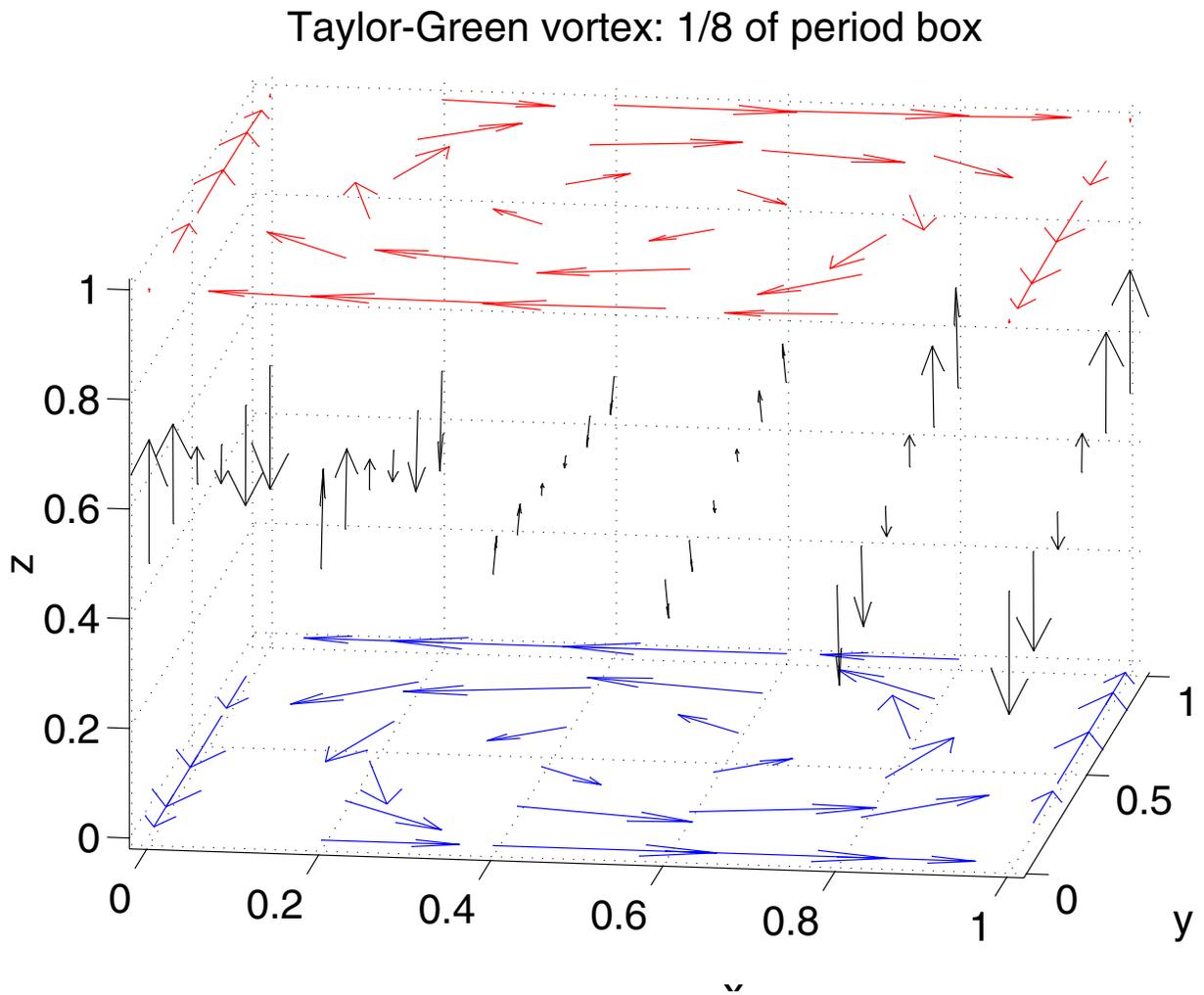


Figure 6.7: A three-dimensional vector field, the initial velocity for the Taylor-Green flow of incompressible, homogeneous turbulence theory. The components are  $u = \sin(\pi x) \cos(\pi y) \cos(\pi z)$ ,  $v = -\cos(\pi x) \sin(\pi y) \cos(\pi z)$ ,  $w = \sqrt{\frac{2}{3}} \cos(\pi x) \cos(\pi y) \sin(\pi z)$ .