TOPOLOGY OPTIMIZATION OF STRUCTURAL SUPPORTS FOR MEMS SWITCHES USING DISCRETE SIMULATED ANNEALING

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ABSTRACT
Micro mechanical (MEMS) switches have sought for many promising applications due to its high power handling capability and low insertion loss compared to the solid-state counterparts. Despite these technological advantages, their reliability still requires much improvement. One of the dominant failure modes of such MEMS switches is buckling due to residual and thermal stresses. As a solution to prevent the buckling, this paper proposes the use of elastic supports that allow switch expansion. Hybrid Discrete Simulated Annealing (HDSA), DSA followed by uni-variant search, is used for finding the optimum topology of the supports for the fixed-fixed beam type MEMS switches using nonlinear finite elements.

KEYWORDS
Topology optimization, MEMS, discrete simulated annealing, uni-variant search, structural analysis

1. INTRODUCTION
Micro-electro mechanical systems (MEMS) are now found in different applications such as medical instruments, hearing aids, air-bag sensors, micro antennas and micro switches for radio-frequency applications [1-3]. Radio frequency (RF) MEMS switch is manufactured using thin film deposition micromachining technique [4]. A typical switch **Fig. 1** is made of a thin beam (typically made from polysilicon or gold) rigidly supported at the two ends. The switch operation is realized by pulling down the beam with electrostatic force. Residual stresses arise in the beam during the manufacturing process. In addition, as the current passing in the beam causes the temperature raise and hence thermal stresses in the beam. Another cause of the thermal stresses is the difference between the thermal expansion coefficients of the beam and the substrate [5]. As a result, the switch can be subject to buckling or creep failure. It was found, for the switch configuration in **Fig. 1**, that buckling due to thermal stress is the dominant failure mode next to the local melting of contact surfaces [6].

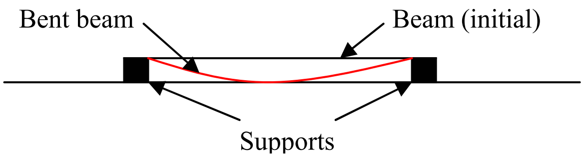


Fig. 1. a typical "fixed-fixed beam" MEMS switch (side view).

This paper proposes a solution to prevent the buckling failure of the fixed-fixed beam type MEMS switches. The main idea is to find an optimum elastic support, instead of the traditional fixed rigid supports, that absorbs the thermal deformation and buckling forces, thereby preventing the buckling. The continuum topology optimization problem using Hybrid Discrete Simulated Annealing (HDSA), DSA followed by discrete uni-variant search, is formulated to find the optimal topology of the support.
The paper starts with a brief review on topology optimization followed by a detailed description of the optimization problem formulation. Afterwards, a description of the HDSA algorithm is given. Finally, the optimization results are presented and discussed.

2. CONTINUUM TOPOLOGY OPTIMIZATION

Topology optimization is often referred to as “Layout Optimization” (or Generalized Shape Optimization). The importance of this type of optimization lies in the fact that the choice of the appropriate topology of a structure in the conceptual phase is generally the most decisive factor for the efficiency of a novel product. The term topology is derived from the Greek word *topos*, which means location, place, space or domain [7].
The first step in the field of continuum topology optimization was by Rossow and Taylor, [8]. Topology optimization of continua was clearly demonstrated by Cheng and Olhoff [9, 10], on optimal thickness distribution for elastic plates. Their work led to a series of works on optimal design problems introducing microstructures in the formulation of the problem [11]. The homogenization method for topology design can be seen as a natural continuation of these studies and has led to the capability to predict computationally the optimal topologies of continuum structures [12]. Topology optimization is now applied extensively in the design MEMS [2, 13].

3. PROBLEM FORMULATION

Table 1. Properties of a typical MEMS switch for radio-frequency (RF) application [6].

Young 's Modulus (Pa)	8.00E+10
Thermal expansion coefficient	1.42E-05
Poisson’s Ratio	0.42
Beam-length “L” (μm)	400
Beam-thickness “t” (μm)	0.5
Beam-width “b” (μm)	50
Temp raise (^oC)	280

Table 1 shows the material properties (Gold) and the dimensions of a typical MEMS switch for radio-frequency (RF) application [6]. Using the straightforward continuum mechanics equations 1, 2 and 3 [14] and the given data, the buckling force and thermal force in the beam as well as the expected deformations due to these forces were calculated as shown in **Table 2**.

Deformation due to a force f: $$\Delta L = \frac{L}{Ebt} f \quad (1)$$

Fixed-fixed buckling load: $$f = \frac{\pi^2 Ebt^3}{3L^2} \quad (2)$$

Elongation due to temperature rise: $$\Delta L = L\alpha\Delta T \quad (3)$$

Table 2. Expected deformations and forces in the MEMS switch in Table 1.

Bucking Force (μN)	10.28
Deformation to avoid buckling (μm)	0.00206
Elongation due to temp. rise (μm)	1.59
Force induced by such elongation (μN)	7952

It can be deduced that the buckling force is too small in comparison to the thermal force induced. As a result one can neglect the buckling force, and our objective at this point would be deigning a support that can absorb thermal deformation giving zero stresses in the beam.

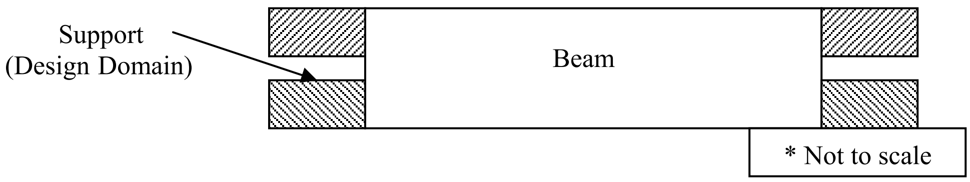


Fig. 2. Location of the supports for the MEMS switch (top view).

Making use of the double symmetry in the problem and ignoring out of plane component of the thermal force (the force angle is 1.1458^{o}), the design domain is reduced to the one shown in **Fig. 3**.

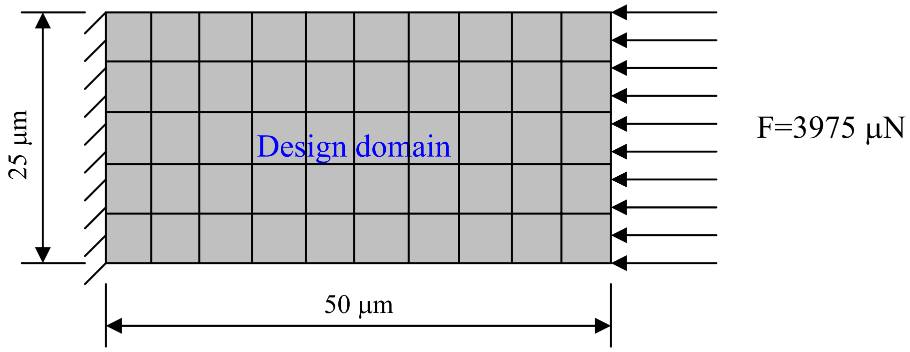


Fig. 3. Discretized design domain for the support.

The domain is discretized into fifty elements (10×5). The optimization was solved as a discrete problem with the discrete density (ρ) of the material at each finite element being a design variable (0 = absent, 1 = present). To overcome re-meshing, absent elements are assumed to have Elasticity modulus of 1/1000 of that of a present element. The total force is redistributed over the present elements at the right edge of the design domain. Since the desired strain in design domain is relatively large ($\varepsilon > 0.01$), nonlinear incremental solver is used to account for the large deformation. The number of increments used was 20.

The objectives are 1) to maximize the average connectivity between any two adjacent columns of elements and 2) to minimize the number of elements used. In other words the problem is formulated as a multi-objective optimization problem:

$$\min_{\rho} : f(\rho) = \frac{-w_1 \sum_{j=1}^{9} \sum_{i=1}^{5} \rho_{i,j} \rho_{i,j+1}}{9} + w_2 \sum_{j=1}^{10} \sum_{i=1}^{5} \rho_{i,j} \tag{4}$$

Subject to:

G1 ≡ Connectivity between any two adjacent columns,

$$G2_i \equiv \text{Deflection constraint} \equiv \Delta_{\max} - |\delta_i(\rho)| \geq 0, \quad i = 1, 2, \ldots, m, \tag{5}$$

$$G3_i \equiv \text{Deflection constraint} \equiv \Delta_{\min} - |\delta_i(\rho)| \le 0, \quad i = 1,2,\ldots,m, \tag{6}$$

$$\rho_{i,j} \in \{0,1\} \tag{7}$$

Where $\rho_{i,\,j}$ is the density of the element in row i and column j, m is the number of present nodes on which the force is applied. w_1 and w_2 are weighing values to ensure equal effect of the two objectives (w_1 = 0.9 and w_2 = 0.1). Δ_{max}, Δ_{min} are the maximum (1.2 μm), minimum (0.8 μm) nodal displacements at the points of loading respectively.
The penalized objective function $f(\rho)$ is now calculated as follows:

$$f(\rho) = \begin{cases} 1000 \times \text{No. of violations of G1} & \text{If G1 is violated} \\ f(\rho) + 1000\sum_{i=1}^{m} G2_i + 1000\sum_{i=1}^{m} G3_i & \text{Otherwise.} \end{cases} \tag{8}$$

Where $G2_i$ and $G3_i$ are equal to zero if their constraints are not violated.

The deflections associated with constraints $G2_i$ and $G3_i$ were evaluated using non-linear finite elements incremental solver. The solver takes into account the thermal expansion due to temperature rise [15, 16].

4. OPTIMIZATION METHOD

To effectively overcome the multi-modal and discrete nature of the topology optimization problem in Section 3, a hybrid optimization approach was used implementing Discrete Simulated Annealing (DSA) method followed by a discrete uni-variant search [17, 18].
Simulated annealing, also known as *Monte Carlo annealing, statistical cooling, probabilistic hill climbing, stochastic relaxation or probabilistic exchange algorithm*, was first introduced by Kirkpatrick *et al.* [19]. This algorithm is known as simulated annealing, due to the analogy with the simulation of the *annealing of solids* it is based upon [20]. The HDSA algorithm starts with the basic DSA algorithm followed by discrete uni-variant search, as illustrated in the following pseudo code [17, 18]:

Begin
Step 1: (Initialization)
- Choose a start point $\underline{\boldsymbol{x}}_{(0,0)}$ for the independent variable.
- Choose a start temperature $T_{(0)} = T_0$
- Set $\underline{\boldsymbol{x}}^* = \underline{\tilde{\boldsymbol{x}}} = \underline{\boldsymbol{x}}_{(0,0)}$, $k = 0$, and $l = 1$

Step 2: (Metropolis simulation)
- Let $\underline{\boldsymbol{x}}_{(k,\,l)} = \text{abs}(\underline{\tilde{\boldsymbol{x}}} + \underline{\boldsymbol{z}})$; Where $\underline{\boldsymbol{z}}$ is a set of discrete random numbers for all independent variables drawn from a the set {0, -1}.
- If $F(\underline{\boldsymbol{x}}_{(k,\,l)}) < F(\underline{\boldsymbol{x}}^*)$, set $\underline{\boldsymbol{x}}^* = \underline{\boldsymbol{x}}_{(k,\,l)}$
- If $F(\underline{\boldsymbol{x}}_{(k,\,l)}) < F(\underline{\tilde{\boldsymbol{x}}})$, go to step 4

  Else

  Draw a uniform random number $p \in [0,1)$

  If $p < \exp\left\{ \dfrac{F(\underline{\tilde{\boldsymbol{x}}}) - F(\underline{\boldsymbol{x}}_{(k,l)})}{T_{(k)}} \right\}$, go to step 4

Step 3: (check the number of Metropolis simulations)
- If $l \ge L_N$, go to step 5.

Step 4: (Inner loop)
- Set $\tilde{\underline{\boldsymbol{x}}} = \underline{\boldsymbol{x}}_{(k,\,l)}$, $l = l + 1$, and go to step 2

Step 5: (termination criterion)
- If $k \geq K_f$, end the DSA and go to Step 7 with solution $\underline{\boldsymbol{x}}^*$

Step 6: (Cooling loop)
- Set $\underline{\boldsymbol{x}}_{(k+1,\,0)} = \underline{\boldsymbol{x}}^*$, and $\tilde{\underline{\boldsymbol{x}}} = \underline{\boldsymbol{x}}^*$
- Cool T; $T_{(k+1)} = \alpha T_{(k)}$, $0 < \alpha < 1$
- Set $l = 1$, $k = k + 1$, and go to step 2

Step 7: (discrete uni-variant search)
- Randomly choose an element to change its state
- If all the elements are chosen, End
- If the element was chosen earlier, repeat step 7
- Set $\rho_{element}$ = abs($\rho_{element}$ - 1)
- Evaluate new(objective function value (OFV))
- If new(OFV) < old (OFV),
  - Update the solution
  - Add the element to the set of chosen elements
  - Repeat step 7

End.

It can be seen that several parameter values, initial temperature value (T_0), number of cooling loops (K_f), length of the Metropolis simulations (L_N), and the decrement of the control parameter (α), must be provided for tuning the algorithm for efficiency.

In the following results, L_N = 1 500, α = 0.7, K_f = 15, leading to a total number of function evaluations = 22 500. The value of T_0 is computed each time the program runs using the following method: For a number of random transitions, calculate the average increase in the objective function value, $\overline{\Delta F}^{+}$

Find T_0 such that:

$$T_0 = \frac{-\overline{\Delta F}^{+}}{\ln(\lambda_0)} \tag{9}$$

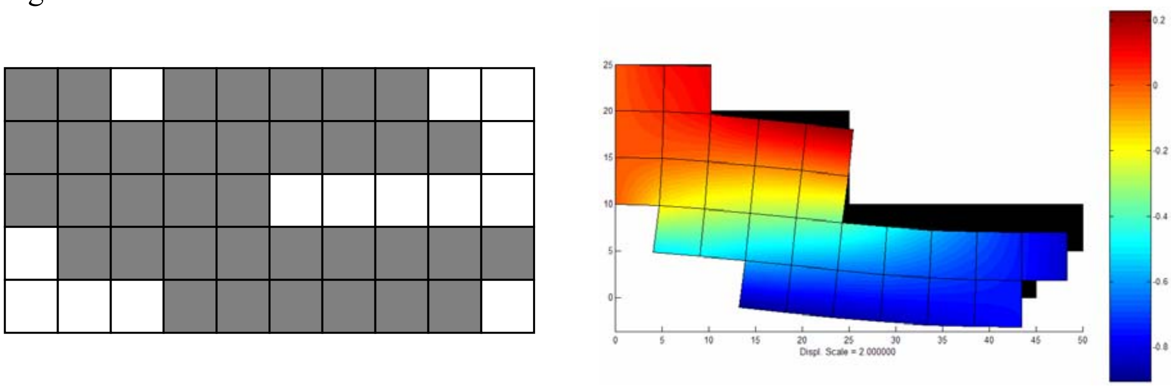
In discrete uni-variant search, all the elements in the mesh (present and absent) are checked randomly. The element density is changed, from 0 to 1 or from 1 to 0, and the objective function is evaluated. If the objective function of the new topology is lower than that of the previous topology, the new topology is accepted. The process is repeated until all the ground structure members are checked.

5. RESULTS AND DISCUSSION

To account for the multi-modality and the random nature of the HDSA algorithm, the problem was solved several times with different starting points. The optimum solution among all HDSA runs is shown in **Fig. 4 (a)**. The redundant elements that were not removed using the HDSA are trimmed manually yielding to the topology shown in **Fig. 4 (b). Table 3** shows the objective function values for DSA, HDSA and the manually trimmed topologies; it also shows the horizontal displacements at the end nodes.

The above topology has the lowest objective function value, the elements are efficiently used and the average connectivity is very high reducing the electrical resistance of the support. The horizontal deflection of the end nodes is not far from the desired deflection (-0.8), which means that the support has an acceptable stiffness as well.

Fig. 5 and **Table 4** shows one of the suboptimal topologies obtained in an optimization run. If it is desired to have smooth, best usage of material, and symmetric support, this topology can be further simplified manually to the one in **Fig. 6** and **Table 5**. It should be noted that these topologies, both before and after manual simplification, are less stiff than the optimum topology in **Fig. 4**. In addition, they have much lower element connectivity which could cause higher electric resistance.

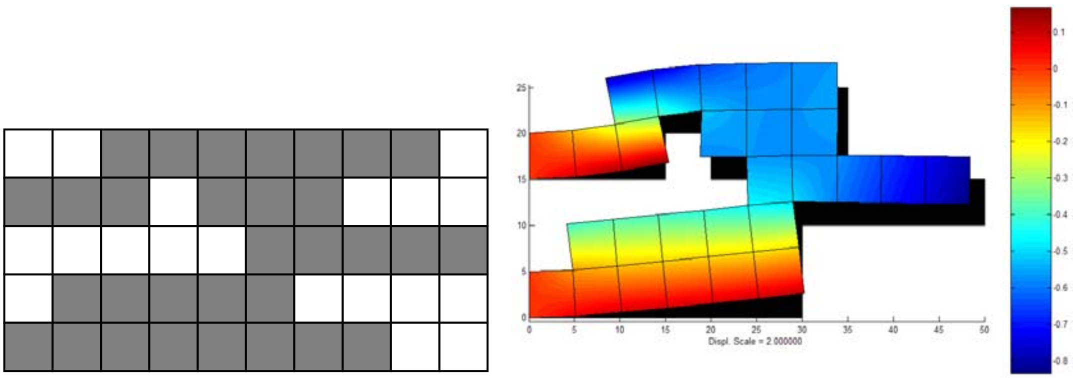


(a) **(b)**

Fig. 4. (a) Optimum output by HDSA; (b) horizontal deformation of the topology after manual trimming.

Table 3. Optimization results and consequent displacements at the end nodes.

	HDSA	Manually trimmed
Objective function value	0.2667	0.2556
Average connectivity	3.3333	2.4444
Average number of elements	3.6000	2.7000
Nodal displacements	-0.8413 -0.8054	-0.8339 -0.8267

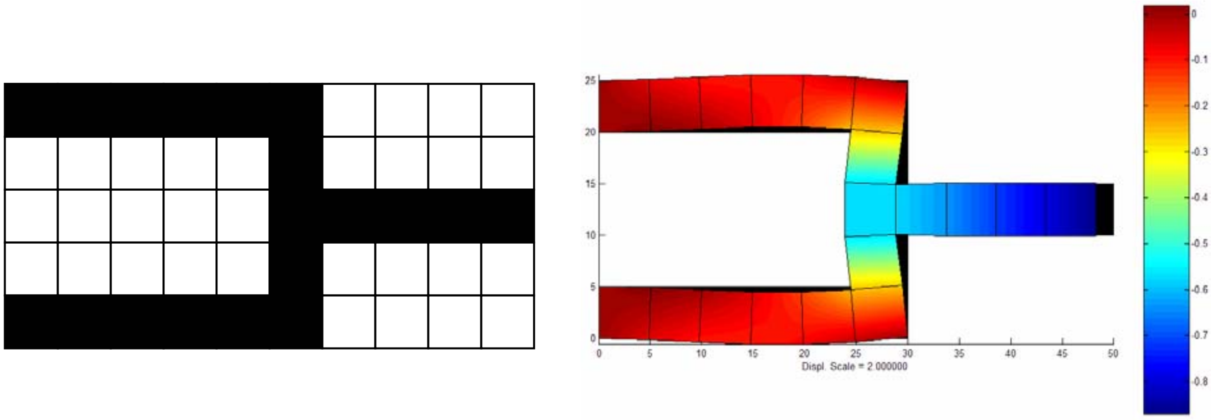


(a) **(b)**

Fig. 5. (a) Suboptimum output by HDSA; (b) horizontal deformation of the topology after manual trimming.

Table 4. Optimization results and consequent displacements at the end nodes.

	HDSA	Manually trimmed
Objective function value	0.3222	0.3667
Average connectivity	2.7778	2.3333
Average number of elements	3.1000	2.7000
Nodal displacements	-0.8045 -0.8070	-0.8323 -0.8026



(a) (b)

Fig. 6. (a) manually simplified topology of Fig. 5 (b) horizontal deformation.

Table 5. Optimization results and consequent displacements at the end nodes.

Objective function value	0.3444
Average connectivity	1.5556
Average number of elements	1.9000
Nodal displacements	-0.8702 -0.8702

6. CONCLUSION

HDSA in addition to manual modification were used to find the optimum topology of structural supports for MEMS switches. Two elastic support designs were presented to increase the thermal reliability of such switches, considering the geometric nonlinearity and thermal expansion.

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