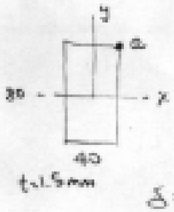
MES99B - winter 2000

1.2 a) $I = th^2\left(\frac{3b+h}{6}\right)$ approximation from **0.2b**



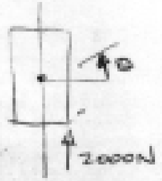
$I_{xx} = (1.5)\,80^2\left[\frac{3\cdot 40+80}{6}\right] = 32\times10^4\ \text{MM}^4$

$\delta = \frac{Fl^3}{3EI} = \frac{(2000N)(1000)^3}{3\cdot 207000\cdot 32\cdot 10^4} = $ **10.06 mm up**

$\sigma_a = \frac{Mc}{I}$ $\quad M = 2000N\ (1000\text{MM})$, $c = \frac{80}{2}\text{MM}$

$\sigma_a = \frac{(2000)\,1000(40)}{32\cdot10^4} = $ **$250\ \frac{N}{MM^2}$ compression**

b) in addition to bending as in a) beam also twists



Torque $T = 2000N\cdot\frac{40}{2}\text{MM}$
$= 4\times10^4\ \text{NMM}$

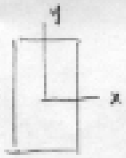
twist $\theta = \frac{TL}{4GA^2}\left(\frac{P}{t}\right)$ where P is perimeter of section

$\theta = \frac{(4\cdot10^4\,\text{NMM})(1000\,\text{MM})}{4\left(79600\frac{N}{MM^2}\right)(40\cdot80\,\text{MM}^2)^2}\cdot\frac{(40+40+80+80)\text{MM}}{1.5\text{MM}}$

$\theta = .002$ RAD $\sim .1145°$

in addition to upward motion from part a), end of beam will also rotate .002 RAD counter clockwise (direct stress is same)

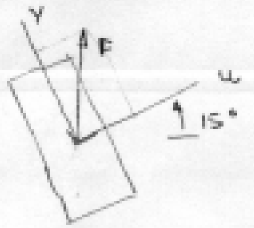
1.2 c)



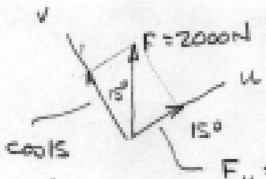
$I_{xx} = 32\times10^4\ \text{MM}^4$ from 1.2a

$I_{yy} = 1.5\ 40^2\left[\frac{3\cdot 80+40}{6}\right] = 11.2\times10^4\ \text{MM}^4$

ROTATED SECTION



$I_u = 32\times10^4\ \text{MM}^4$
$I_v = 11.2\times10^4\ \text{MM}^4$



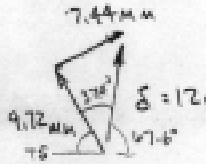
$F_v = 2000\cos 15$
$F_v = 1931.9N$

$F_u = 2000\sin 15$
$F_u = 517.6N$

$\delta_v = \frac{F_v l^3}{3EI_u}$ $\quad \delta_u = \frac{F_u l^3}{3EI_v}$

$\delta_v = \frac{(1931.9)\,1000^3}{3(207000)\,32\cdot10^4}$ $\quad \delta_u = \frac{517.6N\ 1000^3}{3(207000)\ 11.2\cdot10^4}$

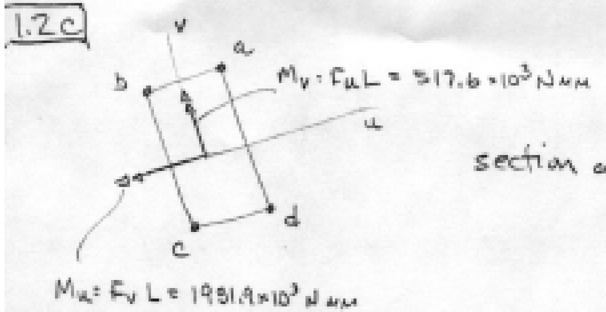
$\delta_v = 9.72\text{MM}$ $\quad \delta_u = 7.44\text{MM}$



$\delta = 12.24\text{MM}$

$\tan^{-1}\frac{7.44}{9.72} = 37.4°$

1.2c



section at fixed end

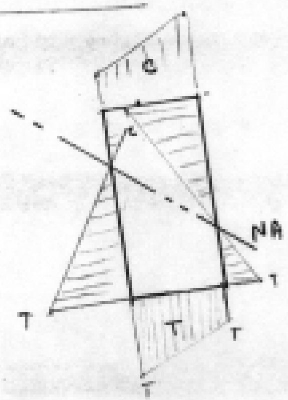
$$\sigma_a = \frac{M_u V_a}{I_u} + \frac{M_v U_a}{I_v} = \frac{(-1931.9 \times 10^3)40}{32 \cdot 10^4} + \frac{-(517.6 \times 10^3)20}{11.2 \cdot 10^4}$$

$$\sigma_a = -241.5 - 92.4 = -333.9 \frac{N}{mm^2} \text{ compression}$$

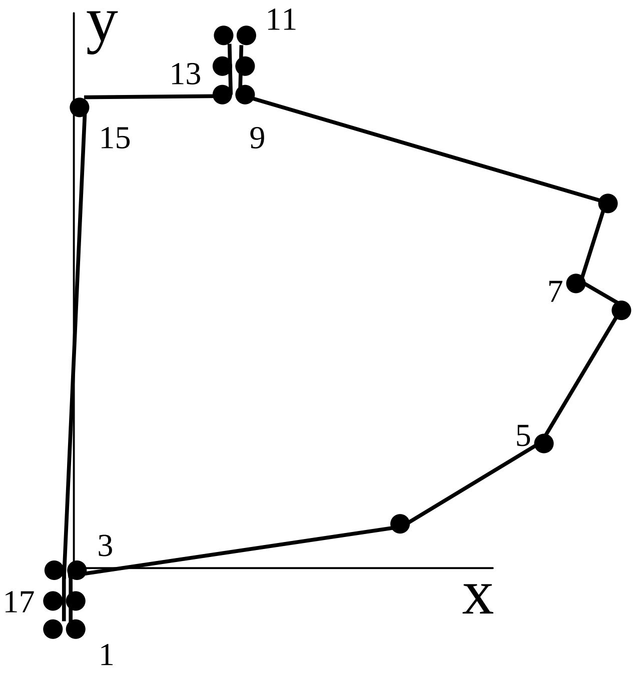
$$\sigma_b = -241.5 + 92.4 = -149.1 \frac{N}{mm^2} \text{ compression}$$

$$\sigma_c = 241.5 + 92.4 = 333.9 \frac{N}{mm^2} \text{ tension}$$

$$\sigma_d = 241.5 - 92.4 = 149.1 \frac{N}{mm^2} \text{ tension}$$



1.6



Cross Section Geometry:

Point No.	X	Y
1	0	-16
2	0	-5
3	0	0
4	77	10
5	112	30
6	130	62
7	120	68
8	127	87
9	45	112
10	45	117
11	45	125
12	44	125
13	44	117
14	44	112
15	3	112
16	-1	0
17	-1	-5
18	-1	-16

Line No.	Start Pt.	End Pt.	Length	Thickness
1	1	2	11	0.62
2	2	3	5	0.62t
3	3	4	77.6466	0.6t
4	4	5	40.3113	0.62
5	5	6	36.7151	0.62t
6	6	7	11.6619	0.62
7	7	8	20.2485	0.62
8	8	9	85.7263	0.62
9	9	10	5	0.62t
10	10	11	8	0.62
11	12	13	8	0.97
12	13	14	5	0.97
13	14	15	41	0.97
14	15	16	112.071	0.97
15	16	17	5	0.97
16	17	18	11	0.97

Weld No.	Start	End
1	2	17
2	10	13

A) Nominal Properties:

Area = 363.42
Ixx = 6.9482E+05
Iyy = 7.5162E+05
Ixy = 1.0271E+05
cx = 43.86
cy = 58.63
Theta = 37.271
Iuu = 6.1666E+05
Ivv = 8.2978E+05
Jc = 8.138E+05

B) Effective Properties:

Area = 222.2

Ixx = 5.0499E+05
Iyy = 5.0608E+05
Ixy = 47140

Theta = 44.666
Iuu = 4.5839E+05
Ivv = 5.5268E+05

C)

	nominal mm^4	effective	
Ixx	6.95E+05	5.05E+05	73%
Iyy	7.52E+05	5.06E+05	67%
Iuu	6.17E+05	4.58E+05	74%
Ivv	8.30E+05	5.53E+05	67%
A	363	222	61%

1.9

$$\sigma_{crit} = \frac{k\pi^2 E}{12(1-\mu^2)\left(\frac{b}{t}\right)^2} = \sigma_y$$

a) $$\frac{b}{t} = \sqrt{\frac{k\pi^2 E}{12(1-\mu^2)\sigma_y}}$$

letting $k=4$ for simply supported edges

$E = 30\cdot10^6$ psi

$\sigma_y = 30\cdot10^3$ psi

$\mu = .3$

b) $$\frac{b}{t} = \sqrt{\frac{4\pi^2 30\cdot10^6}{12(1-.3^2)\,30\cdot10^3}}$$

$$\frac{b}{t} \cong 60$$

so when $\frac{b}{t} < 60$ failure by yield,

$\frac{b}{t} > 60$ failure by buckling

c) $$\frac{b}{t} = \sqrt{\frac{4\pi^2\cdot10\cdot10^6}{12(1-.3^2)\,50\cdot10^3}}$$

$$\frac{b}{t} \cong 27$$

1.11 a

let $b \simeq 90$ MM

$t = .62$

$\frac{b}{t} = 145$

$$\sigma_{CR} = \frac{k\pi^2 E}{12(1-\mu^2)\left(\frac{b}{t}\right)^2} = \frac{4\pi^2(207000\frac{N}{MM^2})}{12(1-.3^2)\,145^2} = 35.6\frac{N}{MM^2}$$

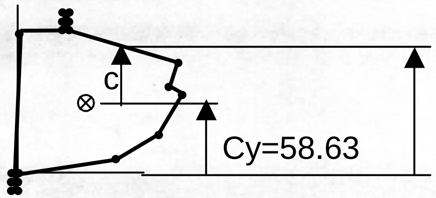
b) $$\sigma = \frac{Mc}{I}$$

$$M = \frac{\sigma I}{c}$$

where I is moment of inertia

c is distance from CG to nominal center of plate

Using results from 1.6 for nominal values: I_{xx}=6.948e5mm^4



To center of plate ~100mm

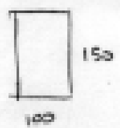
Note: this is a judgement, other values of c on the plate are also ok

c=100-58.63=41.37

$$M = \frac{\sigma I}{c} = \frac{(35.6)(6.948e5)}{41.37} = 6e5\,Nmm$$

1.15

a)



$t = .86$

$$\sigma_{CRIT} = \frac{k\pi^2 E}{12(1-\mu^2)\left(\frac{b}{t}\right)^2} = \frac{4\pi^2\, 207000}{12(1-.3^2)\left(\frac{100}{.86}\right)^2} = 55.35\,\frac{N}{MM^2}$$

$$\sigma = \frac{Mc}{I}$$

$$M = \frac{I\sigma}{c}$$

$\sigma = 55.35\,\frac{N}{MM^2}$

$c = 75\ MM$

$$I = \frac{t h^2 (3b+h)}{6} = \frac{(.86)(150)^2(3\cdot 100 + 150)}{6}$$

$I = 1.451 \times 10^6\ MM^4$

$$M = \frac{(1.451\cdot 10^6)\,55.35}{75} = 1.07\cdot 10^6\ NMM$$

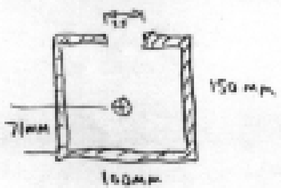
b) using $w = \frac{b}{2}\left(1 + \frac{\sigma_{CRIT}}{\sigma_s}\right)$ other options for w on p 63 of course pack are also OK to u-

$\sigma_s = 1.1\,\sigma_{CR}$ $w = \frac{100}{2}\left(1 + \frac{1}{1.1}\right) = 95.5\ MM$

$\sigma_s = 1.5\,\sigma_{CR}$ $w = \frac{100}{2}\left(1 + \frac{1}{1.5}\right) = 83.3\ MM$

$\sigma_s = 2\,\sigma_{CR}$ $w = \frac{100}{2}\left(1 + \frac{1}{2}\right) = 75\ MM$

c) effective section at $\sigma_s = 2\,\sigma_{CR}$



Compute I_{xx} by hand or use section program

$I_{xx} = 1.32\cdot 10^6\ MM^4$