

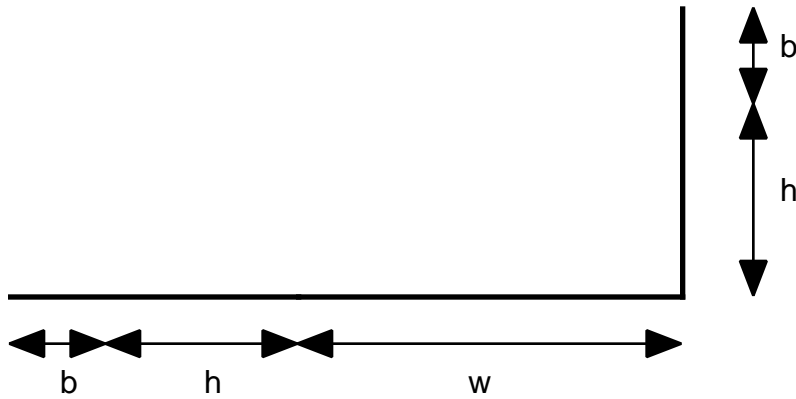
Please submit your answer together with the final examination on April 29. You can look at any notes, handout, and books, but please no consultation to other people.

1. We shall consider a 5 step process of formation of a closed hat section with 1 mm thick sheet metal as follows :

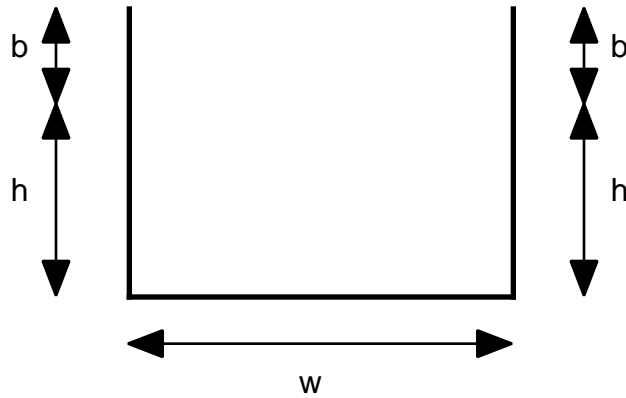
(1)



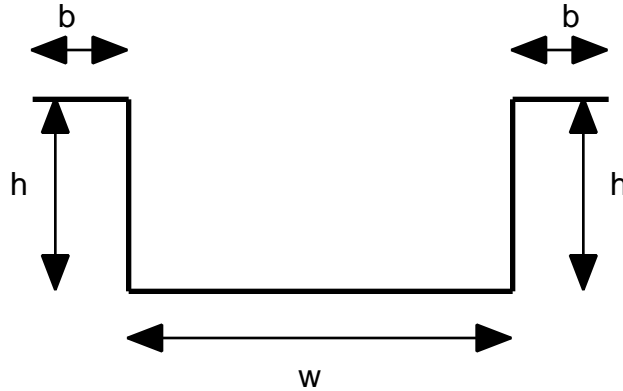
(2)



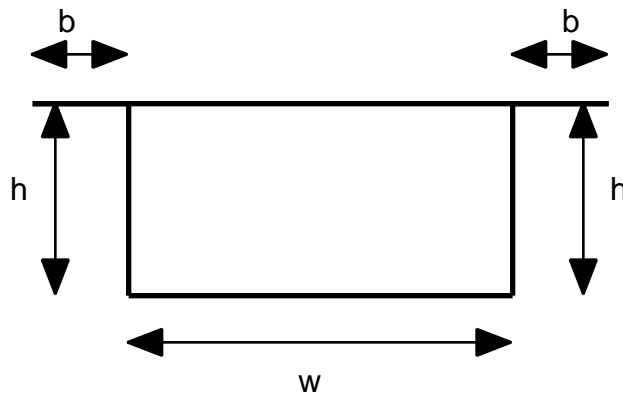
(3)



(4)



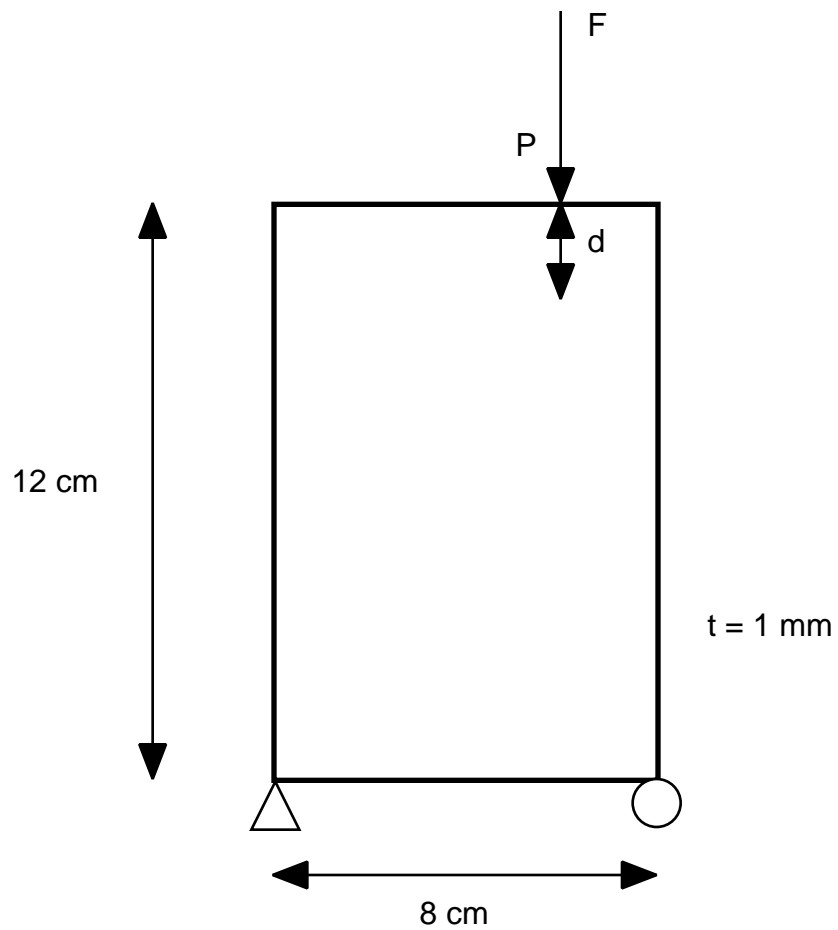
(5)



Here we assume $w = 10$ cm, $h = 5$ cm, and $b = 1$ cm.

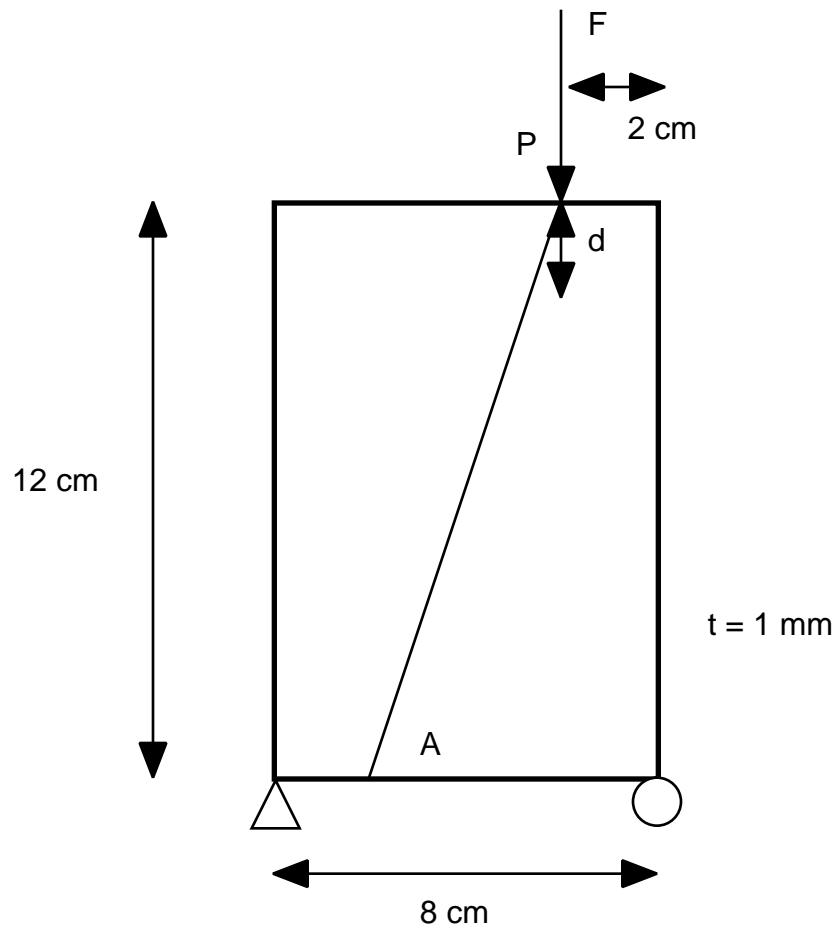
- (a) Find the location of the centroid and shear center of the above five cross sections.
- (b) Find the moment of inertia, torsion constant of the five cross sections, and also the principal axes of the cross sections.
- (c) When the horizontal and vertical forces $f_x = 10$ kN and $f_y = 10$ kN are applied at the centroid of the above five cross sections away from the fixed end of the straight beams with the distance $L = 20$ cm, find the maximum and minimum axial stress σ_z together with the location at the fixed end. It is noted that the x axis is the horizontal direction on the sheet, the y axis is the vertical direction, and the beam axis z is perpendicular to the sheet (i.e. the cross section), while the origin is at the centroid of the cross section. Furthermore, we shall consider only linear elastic deformation, that is, no possibility of plasticity in this problem.
- (d) Find the effective cross sections for the axial compressive load. Tabulate the ratio of the effective portion of the cross section and the original cross section.

2. Applying plane beam elements, we shall develop a finite element model of an idealized cross section of a side member with the unit width :



By adding a thin plate, whose thickness is t_a and the height l_a , to this box beam for reinforcement while the total volume of the additional reinforced plate is kept to be constant, say $V_a = t_a * l_a = 120 \text{ mm}^3/\text{mm}$, find approximately the best location A of the reinforcement so that the local stiffness of the cross section defined by

$$k = \frac{F}{d}$$



can be maximized, where F is an applied vertical force given as 1 kN per unit length (i.e. per 1 mm) at the top of the box beam, and d is the vertical deflection at the loading point P . If necessary, use $E = 200 \text{ Gpa}$, $\nu = 0.3$, and $\kappa = 0$ (shear constant), and it is not necessary to consider the possibility of buckling of “beam” elements in compressive axial stress condition, that is, solve the problem under linear elasticity assumption.