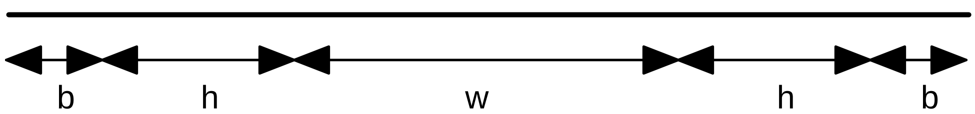
MEAM599-02 **Final Examination (Take Home Part)**
Automotive Body Structures

1998 Winter

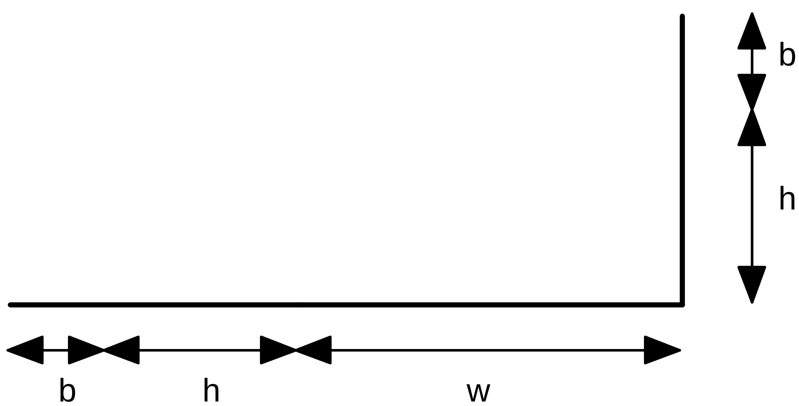
Please submit your answer together with the final examination on April 29. You can look at any notes, handout, and books, but please no consultation to other people.

1. We shall consider a 5 step process of formation of a closed hat section with 1 mm thick sheet metal as follows :

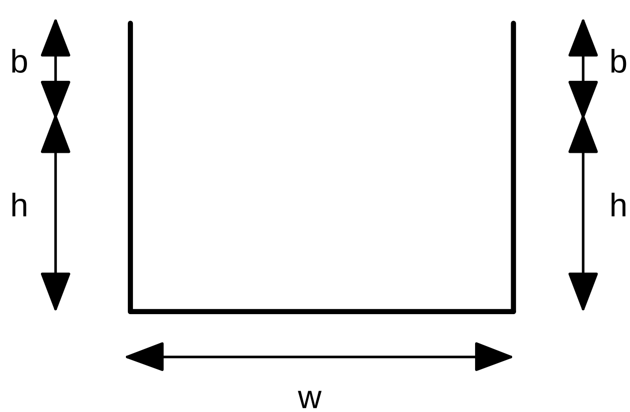
(1)



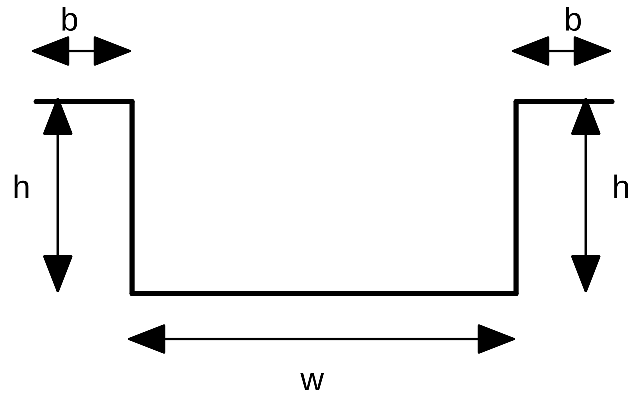
(2)



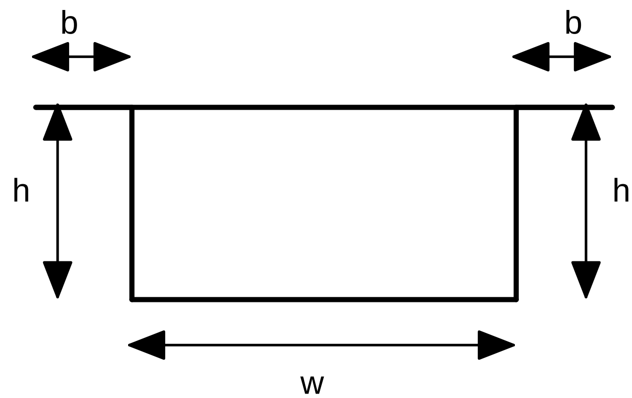
(3)



(4)



(5)



Here we assume w = 10 cm, h = 5 cm, and b = 1 cm.

(a) Find the location of the centroid and shear center of the above five cross sections.

(b) Find the moment of inertia, torsion constant of the five cross sections, and also the principal axes of the cross sections.

(c) When the horizontal and vertical forces f_x = 10 kN and f_y = 10 kN are applied at the centroid of the above five cross sections away from the fixed end of the straight beams with the distance L = 20 cm, find the maximum and minimum axial stress σ_z together with the location at the fixed end. It is noted that the x axis is the horizontal direction on the sheet, the y axis is the vertical direction, and the beam axis z is perpendicular to the sheet (i.e. the cross section), while the origin is at the centroid of the cross section. Furthermore, we shall consider only linear elastic deformation, that is, no possibility of plasticity in this problem.

(d) Find the effective cross sections for the axial compressive load. Tabulate the ratio of the effective portion of the cross section and the original cross section.

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Section 1 :
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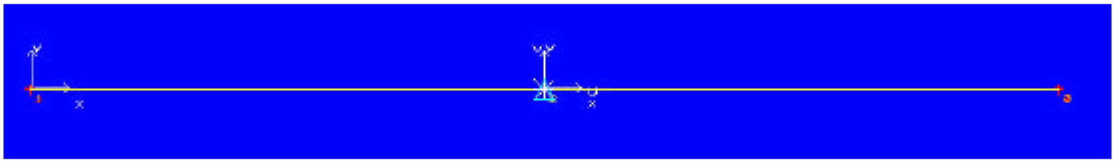


GAS - CARS Geometric Analysis of Sections
Date: Apr 29 1998   Time: 9:35:40
Units   : N, mm, MPa

Cross Section Geometry:

Point No.	X	Y
1	0	0.1
2	0	0
3	220	0
4	220	0.1

Line No.	Start Pt.	End Pt.	Length	Thickness	Material Archive	Material No.	Line Type
1	1	2	0.1	1	asdm.	1	Segment
2	2	3	220	1	asdm.	1	Segment
3	3	4	0.1	1	asdm.	1	Segment

Material Description:

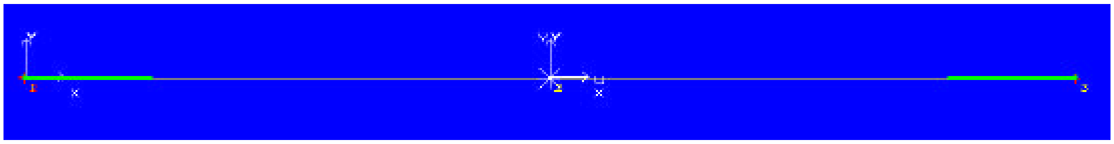
Archive	Mat'l No.	E	Fy	Archive Location
asdm.	1	203000	234.422	c:\cars96\

*** Results ***

Nominal Properties:

Area	=	220.2
cx	=	110
cy	=	4.5413E-05
Ixx	=	18.334
Iyy	=	8.8975E+05
Ixy	=	0
Sx+	=	36.671
Sy+	=	8052.1
Sx-	=	-36.665
Sy-	=	-8052.1
Theta	=	0
Iuu	=	18.334
Ivv	=	8.8975E+05
Su+	=	36.671
Sv+	=	8052.1
Su-	=	-36.665
Sv-	=	-8052.1
rx	=	0.28855
ry	=	63.566
J	=	73.4
Cw	=	8.0514
ex	=	110

ey = -0.00010005
Cuu = 1.5
Cvv = 0.059998
Jopen = 73.4
Jc = 0
tomax = 1
tcmin = 0
Ao = 0



Analysis Option:
Stress Level:
=> Yield Stress
Loading Direction:
=> Axial

*** Results ***

Stresses:

Line No	Stresses Start pt	Stresses End pt
1	234.422	234.422
2	234.422	234.422
3	234.422	234.422

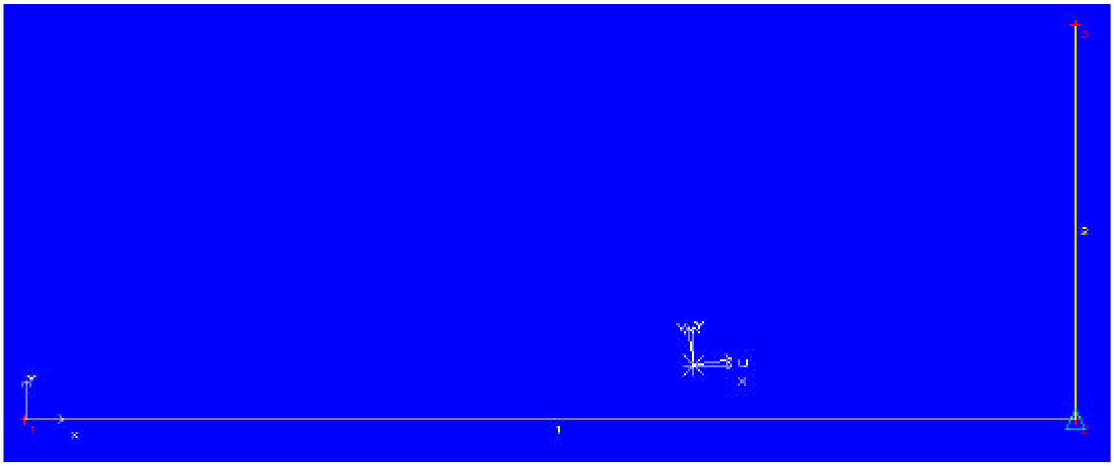
Effective Widths of Compressive Portion of Entities:

Line No.	Compressive Portion	Effective Width	Percent Effective	b1	b2
te					
1	0.1	0.1	100.00	0	0.1
2	220	52.8154	24.01	26.41	26.41
3	0.1	0.1	100.00	0.1	0

Effective Properties:

Area = 53.015
cx = 110
cy = 0.00018862
Ixx = 4.4019
Iyy = 5.0034E+05
Ixy = 0
Sx+ = 8.8072
Sy+ = 4528
Sx- = -8.8006
Sy- = -4528
Theta = 0
Iuu = 4.4019
Ivv = 5.0034E+05
Su+ = 8.8072
Sv+ = 4528
Su- = -8.8006
Sv- = -4528
rx = 0.28815
ry = 97.148

Section 2 :



GAS - CARS Geometric Analysis of Sections
Units : N, mm, MPa

Cross Section Geometry:

Point No.	X	Y
1	0	0
2	0	160
3	60	160

Line No.	Start Pt.	End Pt.	Length	Thickness	Material Archive	Material No.	Line Type
1	1	2	160	1	asdm.	1	Segment
2	2	3	60	1	asdm.	1	Segment

Material Description:

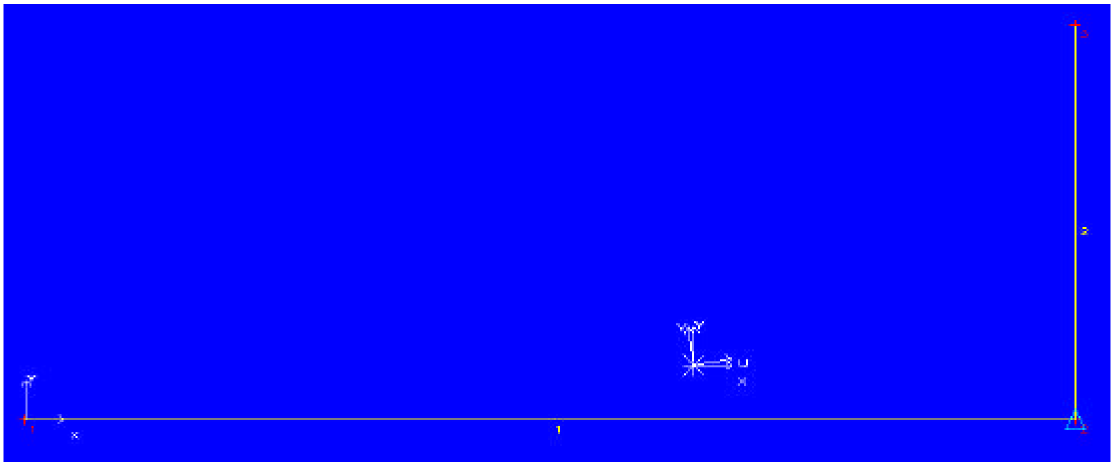
Archive	Mat'l No.	E	Fy	Archive Location
asdm.	1	203000	234.422	c:\cars96\

*** Results ***

Nominal Properties:

Area	=	220
cx	=	8.1818
cy	=	101.82
Ixx	=	6.2061E+05
Iyy	=	57286
Ixy	=	1.0473E+05
Sx+	=	10576
Sy+	=	1105.5
Sx-	=	-6095.3
Sy-	=	-6598.4
Theta	=	-10.198

Iuu	=	6.3945E+05
Ivv	=	38446
Su+	=	9554.1
Sv+	=	942.61
Su-	=	-6284.7
Sv-	=	-2040.1
rx	=	53.113
ry	=	16.137
J	=	73.333
Cw	=	14.291
ex	=	-0.0031349
ey	=	159.98
Cuu	=	3.2114
Cvv	=	1.8063
Jopen	=	73.333
Jc	=	0
tomax	=	1
tcmin	=	0
Ao	=	0



Analysis Option:
 Stress Level:
  => Yield Stress
 Loading Direction:
  => Axial

*** Results ***

Stresses:

Line No	Stresses Start pt	End pt
1	234.422	234.422
2	234.422	234.422

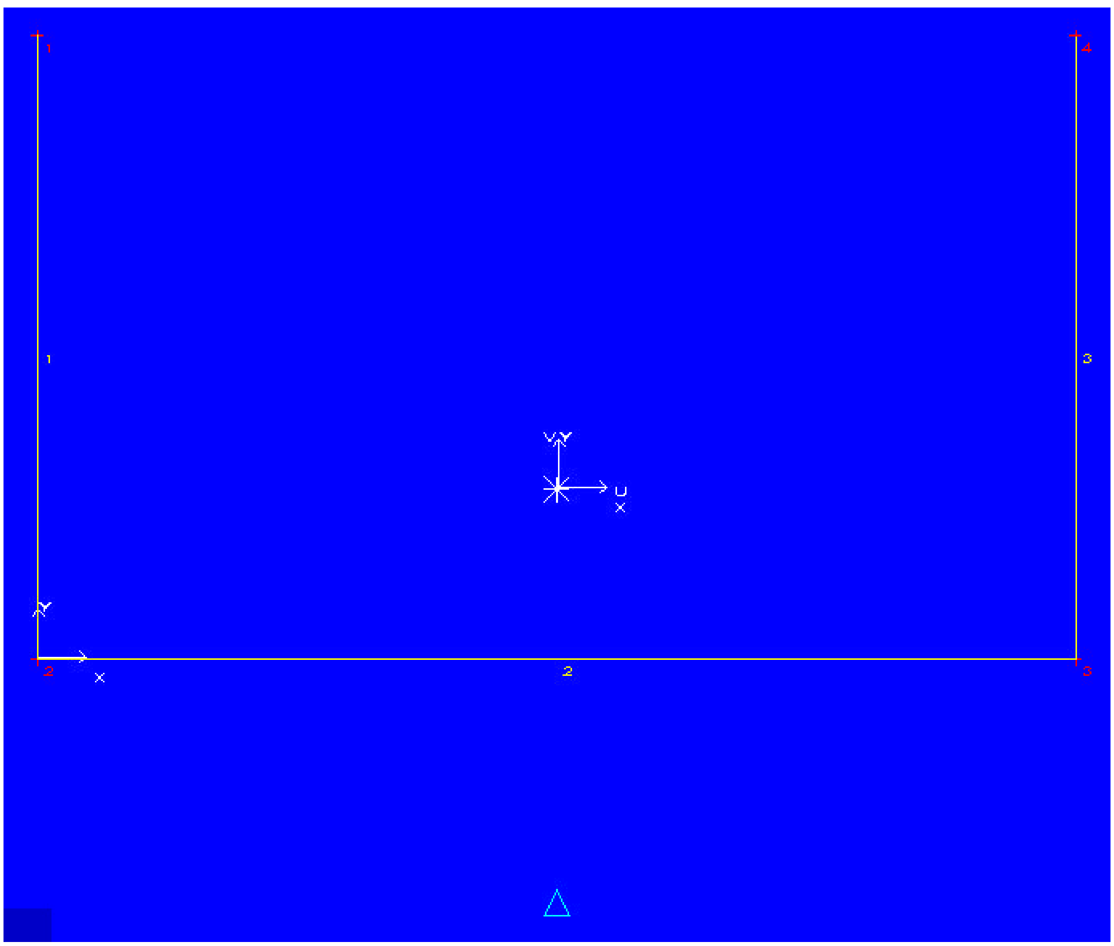
Effective Widths of Compressive Portion of Entities:

Line No.	Compressive Portion	Effective Width	Percent Effective	b1	b2
te					
1	160	17.8802	11.18	0	17.88
2	60	17.1092	28.52	17.11	0

Effective Properties:

```
Area  =       34.989
cx    =        4.183
cy    =       155.43
Ixx   =       1176.6
Iyy   =       1058.7
Ixy   =       668.67
Sx+   =       232.13
Sy+   =       18.967
Sx-   =      -7.5698
Sy-   =      -226.07
Theta =      -42.481
Iuu   =       1788.9
Ivv   =       446.37
Su+   =       43.175
Sv+   =       4.3654
Su-   =      -15.187
Sv-   =      -68.262
rx    =       5.7989
ry    =       5.5006
```

Section 3 :



GAS - CARS Geometric Analysis of Sections
Units : N, mm, MPa

Cross Section Geometry:

Point No.	X	Y

1	0	60
2	0	0
3	100	0
4	100	60

Line No.	Start Pt.	End Pt.	Length	Thickness	Material Archive	No.	Line Type
1	1	2	60	1	asdm.	1	Segment
2	2	3	100	1	asdm.	1	Segment
3	3	4	60	1	asdm.	1	Segment

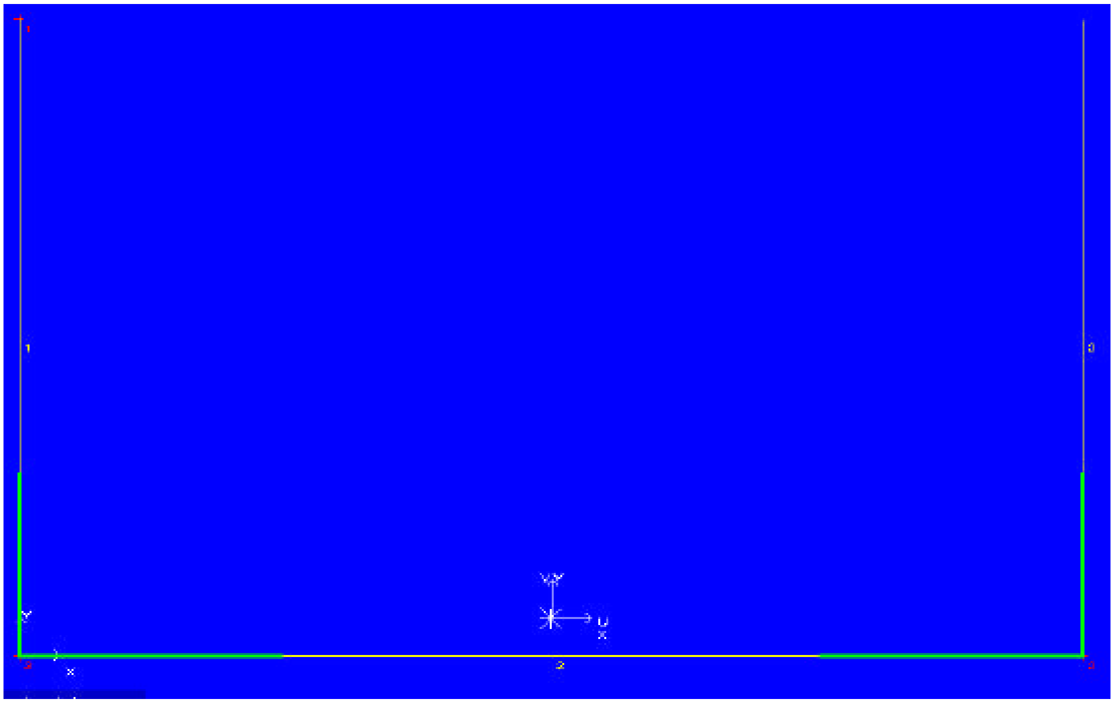
Material Description:

Archive	Mat'l No.	E	Fy	Archive Location
asdm.	1	203000	234.422	c:\cars96\

*** Results ***

Nominal Properties:

Area	=	220
cx	=	50
cy	=	16.364
Ixx	=	85099
Iyy	=	3.8334E+05
Ixy	=	0
Sx+	=	1950.2
Sy+	=	7591
Sx-	=	-5046.3
Sy-	=	-7591
Theta	=	0
Iuu	=	85099
Ivv	=	3.8334E+05
Su+	=	1950.2
Sv+	=	7591
Su-	=	-5046.3
Sv-	=	-7591
rx	=	19.668
ry	=	41.743
J	=	73.333
Cw	=	1.487E+08
ex	=	50
ey	=	-23.477
Cuu	=	2.4391
Cvv	=	2.4613
Jopen	=	73.333
Jc	=	0
tomax	=	1
tcmin	=	0
Ao	=	0



Analysis Option:
 Stress Level:
  => Yield Stress
 Loading Direction:
  => Axial

*** Results ***

Stresses:

Line No	Stresses Start pt	Stresses End pt
1	234.422	234.422
2	234.422	234.422
3	234.422	234.422

Effective Widths of Compressive Portion of Entities:

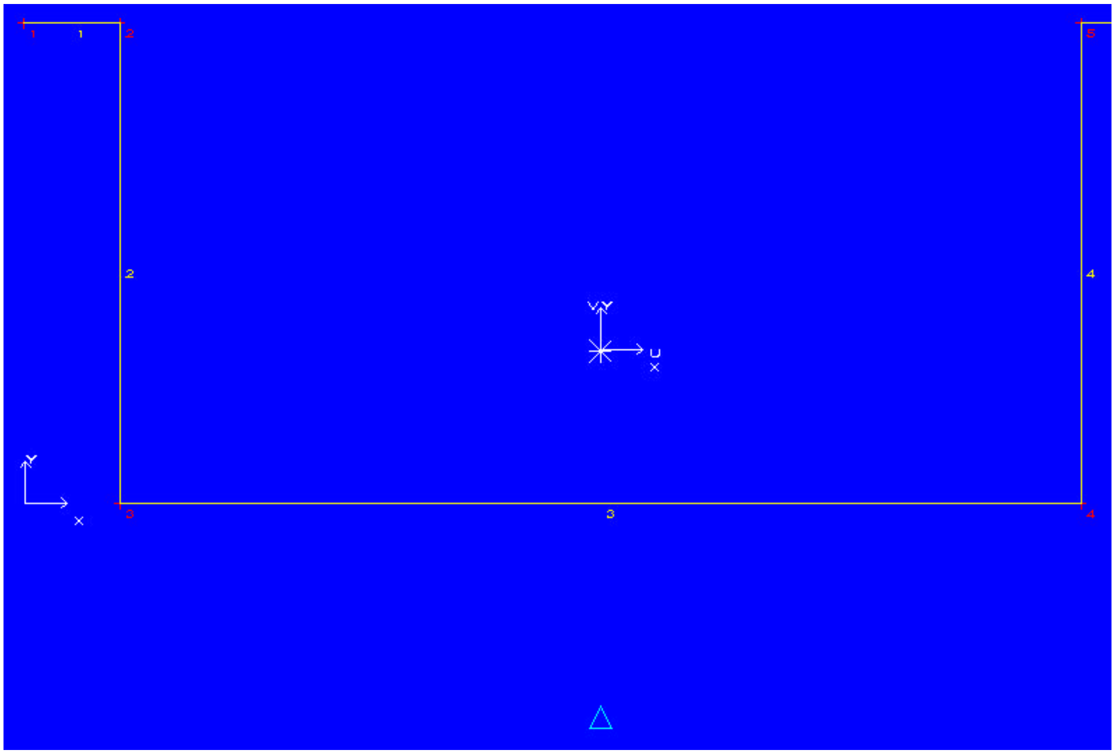
Line No.	Compressive Portion	Effective Width	Percent Effective	b1	b2
te					
1	60	17.1092	28.52	0	17.11
2	100	49.0595	49.06	24.53	24.53
3	60	17.1092	28.52	17.11	0

Effective Properties:

Area	=	83.278
cx	=	50
cy	=	3.515
Ixx	=	2314
Iyy	=	1.5787E+05
Ixy	=	0
Sx+	=	40.967
Sy+	=	3126.1
Sx-	=	-576.34

```
Sy-    =       -3126.1
Theta =              0
Iuu    =          2314
Ivv    =    1.5787E+05
Su+    =        40.967
Sv+    =        3126.1
Su-    =       -576.34
Sv-    =       -3126.1
rx     =        5.2713
ry     =        43.539
```

Section 4:



GAS - CARS Geometric Analysis of Sections
Units : N, mm, MPa

Cross Section Geometry:

Point No.	X	Y
1	0	50
2	10	50
3	10	0
4	110	0
5	110	50
6	120	50

Line No.	Start Pt.	End Pt.	Length	Thickness	Material Archive	No.	Line Type
1	1	2	10	1	asdm.	1	Segment
2	2	3	50	1	asdm.	1	Segment

3	3	4	100	1	asdm.	1
Segment						
4	4	5	50	1	asdm.	1
Segment						
5	5	6	10	1	asdm.	1
Segment						

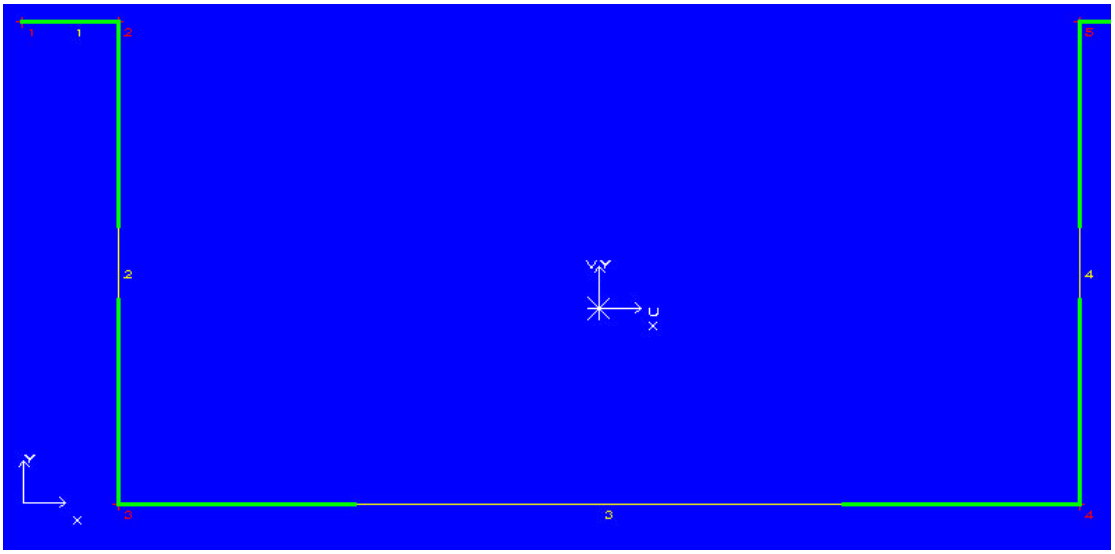
Material Description:

Archive	Mat'l No.	E	Fy	Archive Location
asdm.	1	203000	234.422	c:\cars96\

*** Results ***

Nominal Properties:

Area	=	220
cx	=	60
cy	=	15.909
Ixx	=	77662
Iyy	=	3.9401E+05
Ixy	=	0
Sx+	=	2245.1
Sy+	=	6566.8
Sx-	=	-4732.8
Sy-	=	-6566.8
Theta	=	0
Iuu	=	77662
Ivv	=	3.9401E+05
Su+	=	2245.1
Sv+	=	6566.8
Su-	=	-4732.8
Sv-	=	-6566.8
rx	=	18.788
ry	=	42.32
J	=	73.333
Cw	=	1.1716E+08
ex	=	60
ey	=	-22.123
Cuu	=	2.401
Cvv	=	2.6119
Jopen	=	73.333
Jc	=	0
tomax	=	1
tcmin	=	0
Ao	=	0



Analysis Option:
 Stress Level:
  => Yield Stress
 Loading Direction:
  => Axial

*** Results ***

Stresses:

Line No	Stresses Start pt	Stresses End pt
1	234.422	234.422
2	234.422	234.422
3	234.422	234.422
4	234.422	234.422
5	234.422	234.422

Effective Widths of Compressive Portion of Entities:

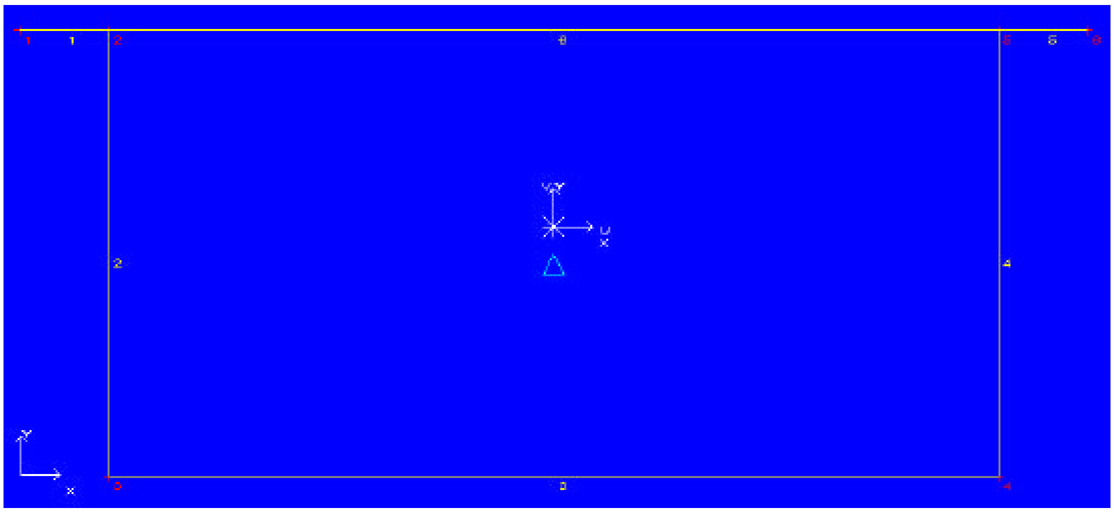
Line No. te	Compressive Portion	Effective Width	Percent Effective	b1	b2
1	10	10	100.00	0	10
2	50	42.1738	84.35	21.09	21.09
3	100	49.0595	49.06	24.53	24.53
4	50	42.1738	84.35	21.09	21.09
5	10	10	100.00	10	0

Effective Properties:

Area	=	153.41
cx	=	60
cy	=	20.264
Ixx	=	60481
Iyy	=	3.4386E+05
Ixy	=	0
Sx+	=	2000.3
Sy+	=	5731
Sx-	=	-2912.7
Sy-	=	-5731

Theta	=	0
Iuu	=	60481
Ivv	=	3.4386E+05
Su+	=	2000.3
Sv+	=	5731
Su-	=	-2912.7
Sv-	=	-5731
rx	=	19.856
ry	=	47.344

Section 5 :



Units : N, mm, MPa

Cross Section Geometry:

Point No.	X	Y
1	0	50
2	10	50
3	10	0
4	110	0
5	110	50
6	120	50

Line No.	Start Pt.	End Pt.	Length	Thickness	Material Archive	Material No.	Line Type
1	1	2	10	2	asdm.	1	Segment
2	2	3	50	1	asdm.	1	Segment
3	3	4	100	1	asdm.	1	Segment
4	4	5	50	1	asdm.	1	Segment
5	5	6	10	2	asdm.	1	Segment
6	5	2	100	1	asdm.	1	Segment

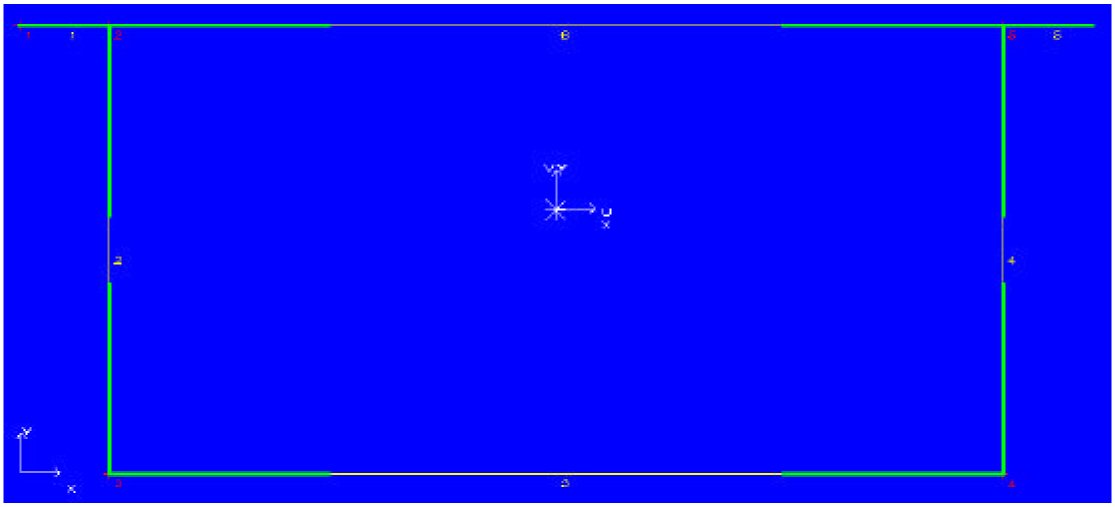
Material Description:

Archive	Mat'l No.	E	Fy	Archive Location
asdm.	1	203000	234.422	c:\cars96\

*** Results ***

Nominal Properties:

```
Area   =         340
cx     =          60
cy     =      27.941
Ixx    =  1.6792E+05
Iyy    =  5.3801E+05
Ixy    =           0
Sx+    =      7282.3
Sy+    =      8966.8
Sx-    =     -5904.2
Sy-    =     -8966.8
Theta  =           0
Iuu    =  1.6792E+05
Ivv    =  5.3801E+05
Su+    =      7282.3
Sv+    =      8966.8
Su-    =     -5904.2
Sv-    =     -8966.8
rx     =      22.224
ry     =      39.779
J      =  3.3339E+05
Cw     =           0
ex     =          60
ey     =      23.823
Cuu    =      1.9275
Cvv    =      3.6191
Jopen  =      53.333
Jc     =  3.3333E+05
tomax  =           2
tcmin  =           1
Ao     =        5000
```



Analysis Option:
 Stress Level:
  => Yield Stress
 Loading Direction:
  => Axial

*** Results ***

Stresses:

Line No	Stresses Start pt	End pt
1	234.422	234.422
2	234.422	234.422
3	234.422	234.422
4	234.422	234.422
5	234.422	234.422
6	234.422	234.422

Effective Widths of Compressive Portion of Entities:

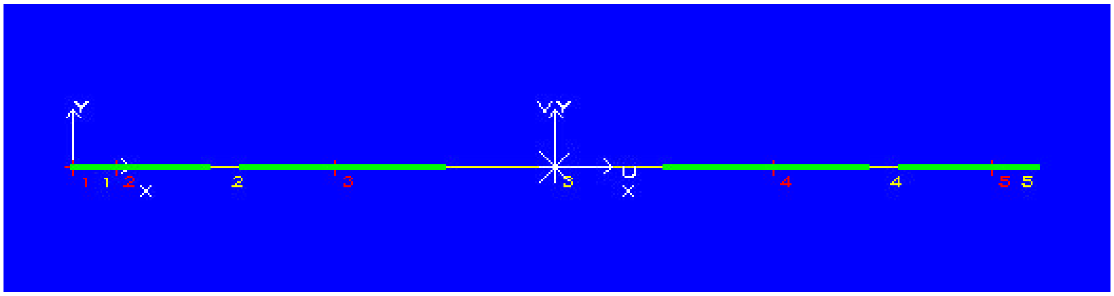
Line No.	Compressive Portion	Effective Width	Percent Effective	b1	b2
te					
1	10	10	100.00	0	10
2	50	42.1738	84.35	21.09	21.09
3	100	49.0595	49.06	24.53	24.53
4	50	42.1738	84.35	21.09	21.09
5	10	10	100.00	10	0
6	100	49.0595	49.06	24.53	24.53

Effective Properties:

```
Area  =      222.47
cx    =          60
cy    =      29.495
Ixx   =   1.026E+05
Iyy   =  4.7684E+05
Ixy   =           0
Sx+   =      4771.2
Sy+   =      7947.4
Sx-   =     -3420.7
Sy-   =     -7947.4
Theta =           0
Iuu   =   1.026E+05
Ivv   =  4.7684E+05
Su+   =      4771.2
Sv+   =      7947.4
Su-   =     -3420.7
Sv-   =     -7947.4
rx    =      21.476
ry    =      46.297
```

<u>One Special Note on CARS'96</u>

I believe CARS'96 has a serious bug when we compute the effective width of the cross section. For example, if we model the flat section, that is, Section 1 in the present problem, by decomposing it into several small pieces, then the effective width is obtained for each small segment, and then the result becomes as follows :



This is clearly different from the one I got in above.

Summary of the Computed Result

s	cx	cy	ex	ey	Iuu	Ivv	θ	J	%
1	110	0	110	0	1.80E+01	8.90E+05	0	7.30E+01	26.1
2	102	8.2	160	0	3.80E+04	6.40E+05	10.2	7.30E+01	15.7
3	50	16.4	50	-23.5	8.50E+04	3.80E+05	0	7.30E+01	35.7
4	60	15.9	60	-22.1	7.80E+04	3.90E+05	0	7.30E+01	69.1
5	60	27.9	60	23.8	1.70E+05	5.40E+05	0	3.30E+05	62.5

Stress Distribution

We shall apply the elementary beam theory :

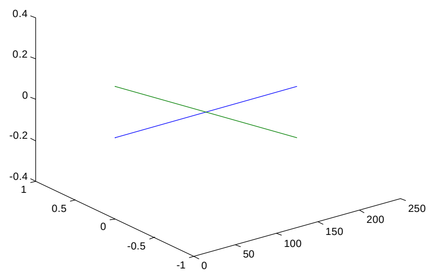
$$\sigma_z = -\frac{M_u}{I_{uu}} v - \frac{M_v}{I_{vv}} u$$

where u and v are the distance of a point on the cross section of the beam from the centroid of the cross section in the principal axis direction, respectively.

Section 1

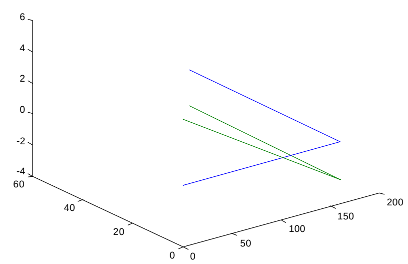
Mu = 2000 and Mv = 2000

xs = -110 110
ys = 0 0
sz =-0.2473



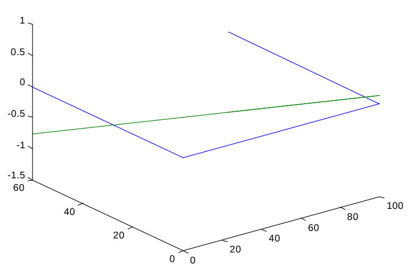
Section 2

xs = -26.0833 2.2503 61.3020
ys = 98.7619 -58.7094 -48.0843
sz = 4.2414 -2.4732 -2.2416



Section 3

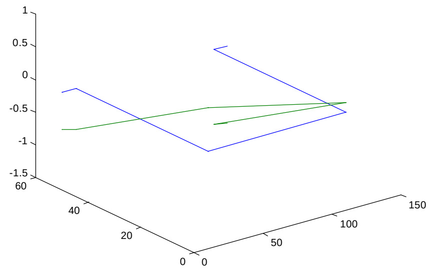
xs = -50 -50 50 50
ys = 43.6360 -16.3640 -16.3640 43.6360
sz = -0.7647 0.6455 0.1237 -1.2864



Section 4

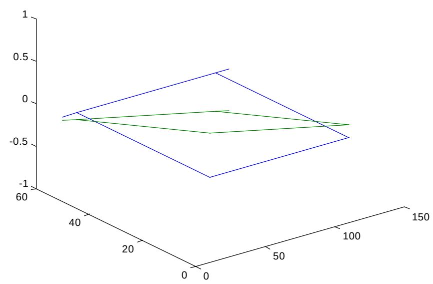
xs = -60 -50 -50 50 50 60

ys =   34.0910   34.0910 -15.9090 -15.9090   34.0910   34.0910
sz =   -0.5734   -0.6241   0.6635   0.1559   -1.1317   -1.1825

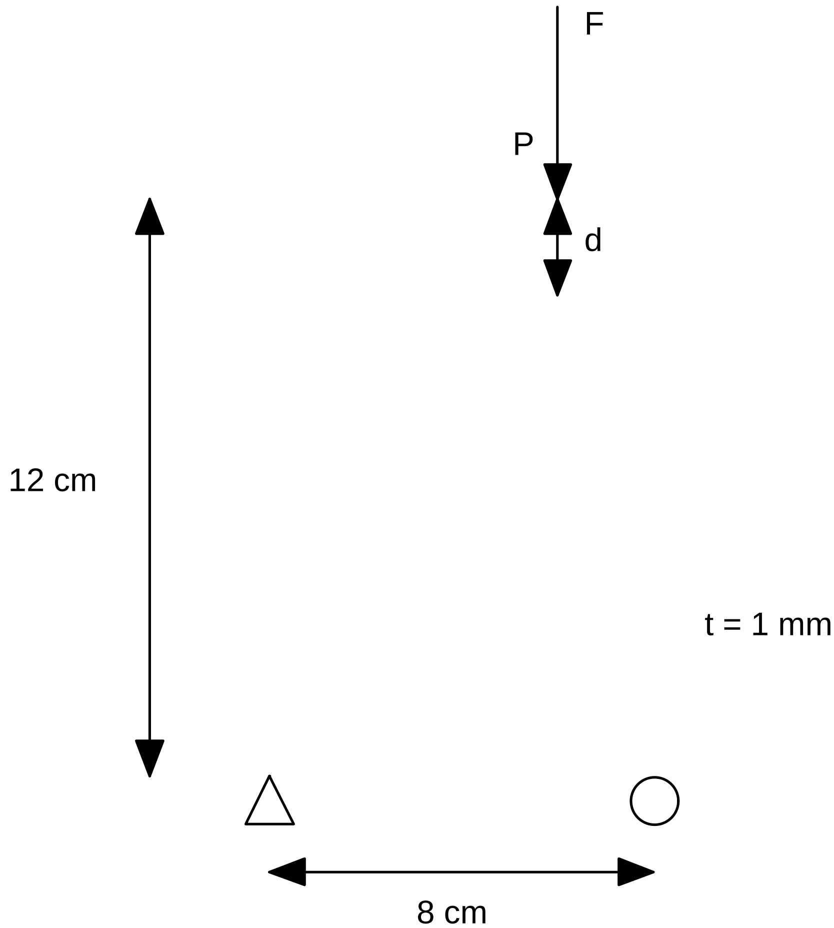


Section 5

xs =   -60   -50   -50   50   50   60
ys =   22.0590   22.0590 -27.9410 -27.9410   22.0590   22.0590
sz =   -0.0397   -0.0769   0.5187   0.1469   -0.4486   -0.4858

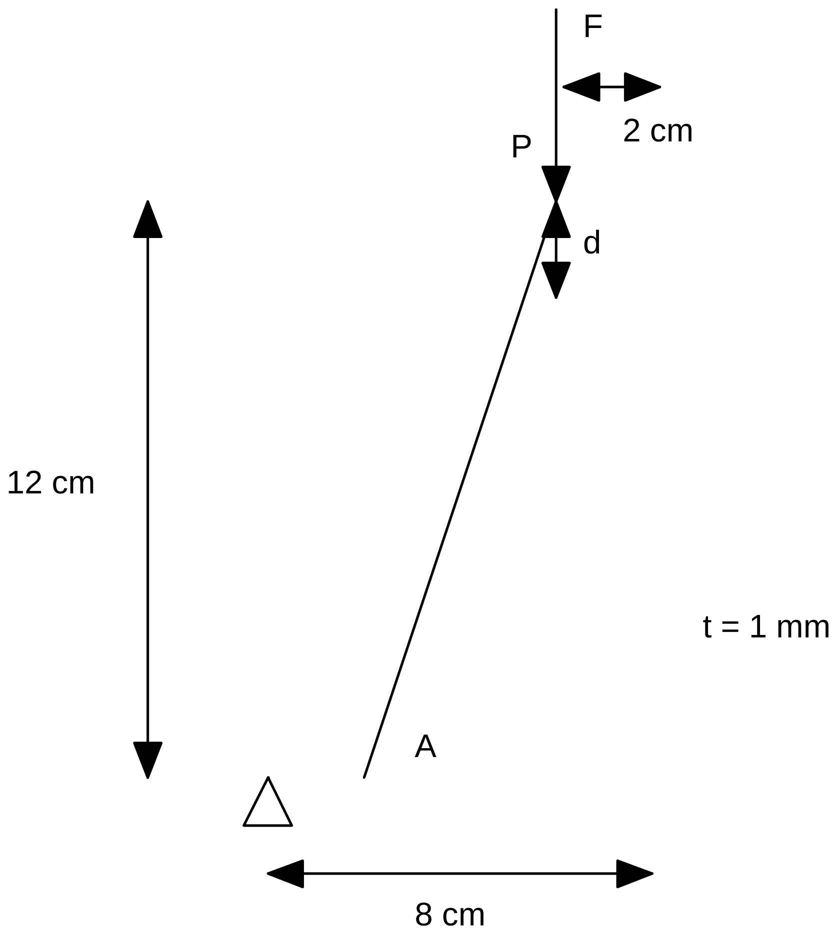


2. Applying plane beam elements, we shall develop a finite element model of an idealized cross section of a side member with the unit width :



By adding a thin plate, whose thickness is t_a and the height l_a, to this box beam for reinforcement while the total volume of the additional reinforced plate is kept to be constant, say $V_a = t_a * l_a = 120$ mm^3/mm, find approximately the best location A of the reinforcement so that the local stiffness of the cross section defined by

$$k = \frac{F}{d}$$



can be maximized, where F is an applied vertical force given as 1 kN per unit length (i.e. per 1 mm) at the top of the box beam, and d is the vertical deflection at the loading point P. If necessary, use E = 200 Gpa, ν = 0.3, and Φ = 0 (shear constant), and it is not necessary to consider the possibility of buckling of “beam” elements in compressive axial stress condition, that is, solve the problem under linear elasticity assumption.

Case 1 : Point A is assumed on the bottom line

Using the following MATLAB program

```
% MATLAB Program : Take Home Final Examination - 2(1)
%
% MEAM 599- 02 / 1998 Winter
% FEM for Side Frame Analysis using plane beam elements
% with possibly flexible joints
%
% is modified for Take Home Final Examination
%
% ________________________________________________________________
%
result=[];
%
```

```
for diter=1:9
   z4=(diter-1)*10;
   if diter==1, z4=1;, end
   if diter==9, z4=79;, end
% Pre-Processing / set up an analysis model
%
% read nodal coordinates ( Z, Y )
nx=8;
Z=[0,0,0,z4,80,80,80,60];
Y=[120,60,0,0,0,60,120,120];
% plot the nodes of the plane beam structure
plot(Z,Y,'+')
% read element connectivity and section type of beam elements
%     ijk(1,nel)=node i of beam element nel
%     ijk(2,nel)=node j of beam element nel
%     ijk(3,nel)=section type of beam element nel
nelx=9;
ijk=[1,2,1;2,3,1;3,4,1;4,5,1;5,6,1;6,7,1;7,8,1;8,1,1;8,4,2]';
% plot the beam elements
for nel=1:nelx
   Ze(2*nel-1)=Z(ijk(1,nel));
   Ze(2*nel)=Z(ijk(2,nel));
   Ye(2*nel-1)=Y(ijk(1,nel));
   Ye(2*nel)=Y(ijk(2,nel));
end
plot(Z,Y,'+',Ze,Ye)
% read section type ( properties )
nsecx=2;
Ei=200000;
la=sqrt((Z(8)-Z(4))^2+(Y(8)-Y(4))^2);
t=120/la;
Iyy1=1/12;
Iyy2=t^3/12;
E=Ei*[1,1];
A=[1,t];
Iyy=[Iyy1,Iyy2];
Fy=[0,0];
% displacement constraints
spc=[];
nspc=3;
spc=[3,1,0;3,2,0;5,1,0]';
% applied forces and moments at the nodes
afm=[];
nafm=1;
afm=[8,1,-1000]';
%
%
% FE-Processing / forming the global stiffness matrix
%
% beam elements
%
sk=zeros(3*nx);
f=zeros(3*nx,1);
sksize=3*nx;
%
for nel=1:nelx
%
   Zji=Z(ijk(2,nel))-Z(ijk(1,nel));
   Yji=Y(ijk(2,nel))-Y(ijk(1,nel));
   Lji=sqrt(Zji^2+Yji^2);
   cji=Zji/Lji;
   sji=Yji/Lji;
   TG=zeros(6);
   TG(1,1)= cji;
```

```
   TG(1,2)=-sji;
   TG(2,1)= sji;
   TG(2,2)= cji;
   TG(3,3)= 1;
   TG(4,4)= cji;
   TG(4,5)=-sji;
   TG(5,4)= sji;
   TG(5,5)= cji;
   TG(6,6)= 1;
%
   skel=zeros(6);
   ijk3=ijk(3,nel);
   EIyy=E(ijk3)*Iyy(ijk3);
   Fy1=(1+Fy(ijk3));
   skel(1,1)=12*E(ijk3)*Iyy(ijk3)/((1+Fy(ijk3))*Lji^3);
   skel(2,2)=E(ijk3)*A(ijk3)/Lji;
   skel(3,1)=-6*E(ijk3)*Iyy(ijk3)/((1+Fy(ijk3))*Lji^2);
   skel(3,3)=(4+Fy(ijk3))*E(ijk3)*Iyy(ijk3)/((1+Fy(ijk3))*Lji);
   skel(4,1)=-skel(1,1);
   skel(4,3)=-skel(3,1);
   skel(4,4)= skel(1,1);
   skel(5,2)=-skel(2,2);
   skel(5,5)= skel(2,2);
   skel(6,1)= skel(3,1);
   skel(6,3)=(2-Fy(ijk3))*E(ijk3)*Iyy(ijk3)/((1+Fy(ijk3))*Lji);
   skel(6,4)=-skel(3,1);
   skel(6,6)= skel(3,3);
   for i=1:5
      for j=1+1:6
         skel(i,j)=skel(j,i);
      end
   end
%
   skeg=TG'*skel*TG;
%
   ijk1=ijk(1,nel);
   ijk2=ijk(2,nel);
   ndg=[3*ijk1-2,3*ijk1-1,3*ijk1,3*ijk2-2,3*ijk2-1,3*ijk2];
   for i=1:6
      for j=1:6
         sk(ndg(i),ndg(j))=sk(ndg(i),ndg(j))+skeg(i,j);
      end
   end
%
end
%
% single point constraints
%
for i=1:nspc
   spc1=spc(1,i);
   spc2=spc(2,i);
   spc3=spc(3,i);
   ndof=3*(spc1-1)+spc2;
   f=f-sk(:,ndof)*spc3;
   sk(:,ndof)=zeros(sksize,1);
   sk(ndof,:)=zeros(1,sksize);
   sk(ndof,ndof)=1;
end
%
% applied for#es and moments
%
for i=1:nafm
   afm1=afm(1,i);
   afm2=afm(2,i);
```

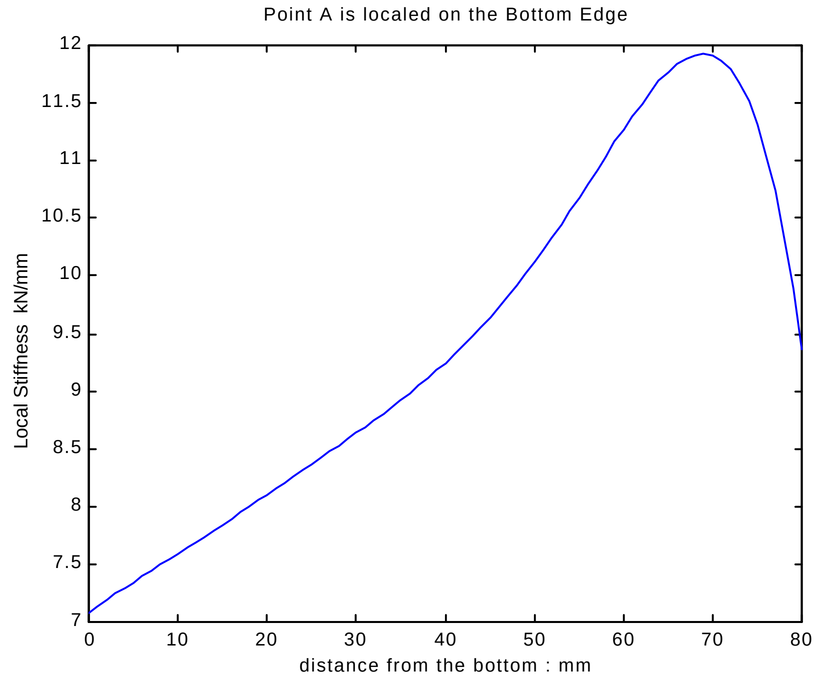
```
    afm3=afm(3,i);
    ndof=3*(afm1-1)+afm2;
    f(ndof)=f(ndof)+afm3;
end
%
% solving the matrix equation
%
d=sk\f;
disp=[];
for i=1:nx
    disp(i,1)=i;
    disp(i,2)=d(3*i-2);
    disp(i,3)=d(3*i-1);
    disp(i,4)=d(3*i);
end
disp;
%
% Post-Processing of the computed results
%
strainenergy=[];
axialstress=[];
Zp=[];
Yp=[];
uZ=[];
uY=[];
for nel=1:nelx
    Zji=Z(ijk(2,nel))-Z(ijk(1,nel));
    Yji=Y(ijk(2,nel))-Y(ijk(1,nel));
    Lji=sqrt(Zji^2+Yji^2);
    cji=Zji/Lji;
    sji=Yji/Lji;
    TG=zeros(6);
    TG(1,1)= cji;
    TG(1,2)=-sji;
    TG(2,1)= sji;
    TG(2,2)= cji;
    TG(3,3)= 1;
    TG(4,4)= cji;
    TG(4,5)=-sji;
    TG(5,4)= sji;
    TG(5,5)= cji;
    TG(6,6)= 1;
    deg=zeros(6,1);
    for i=1:2
        ijki=ijk(i,nel);
        for j=1:3
            deg(3*(i-1)+j)=d(3*(ijki-1)+j);
        end
    end
    del=TG*deg;
    skel=zeros(6);
    ijk3=ijk(3,nel);
    EIyy=E(ijk3)*Iyy(ijk3);
    Fy1=(1+Fy(ijk3));
    skel(1,1)=12*E(ijk3)*Iyy(ijk3)/((1+Fy(ijk3))*Lji^3);
    skel(2,2)=E(ijk3)*A(ijk3)/Lji;
    skel(3,1)=-6*E(ijk3)*Iyy(ijk3)/((1+Fy(ijk3))*Lji^2);
    skel(3,3)=(4+Fy(ijk3))*E(ijk3)*Iyy(ijk3)/((1+Fy(ijk3))*Lji);
    skel(4,1)=-skel(1,1);
    skel(4,3)=-skel(3,1);
    skel(4,4)= skel(1,1);
    skel(5,2)=-skel(2,2);
    skel(5,5)= skel(2,2);
    skel(6,1)= skel(3,1);
```

```
   skel(6,3)=(2-Fy(ijk3))*E(ijk3)*Iyy(ijk3)/((1+Fy(ijk3))*Lji);
   skel(6,4)=-skel(3,1);
   skel(6,6)= skel(3,3);
   for i=1:5
      for j=1+1:6
         skel(i,j)=skel(j,i);
      end
   end
   nel;
   strainenergynel=(del'*skel*del)/2;
   strainenergy(nel)=strainenergynel;
% axial strain  ez = ez0 + y ( ezi*(1-z/l) + ezj*(z/l) )
   ez0=(del(5)-del(2))/Lji;
   ezi= 6*(del(1)-del(4))/Lji^2-(4*del(3)+2*del(6))/Lji;
   ezj=-6*(del(1)-del(4))/Lji^2+(2*del(3)+4*del(6))/Lji;
   axialstrain=[ez0,ezi,ezj];
% axial stress  sz = sz0 + y ( szi*(1-z/l) + szj*(z/l) )
   sz0=E(ijk3)*ez0;
   szi=E(ijk3)*ezi;
   szj=E(ijk3)*ezj;
   axialstress(nel,1)=sz0;
   axialstress(nel,2)=szi;
   axialstress(nel,3)=szj;
% displacement of the beam axis
   ipx=11;
   for ip=1:ipx
      zpi=(ip-1)/(ipx-1);
      nv=[1-3*zpi^2+2*zpi^3,Lji*(zpi-2*zpi^2+zpi^3),3*zpi^2-
2*zpi^3,Lji*(-zpi^2+zpi^3)];
      nw=[1-zpi,zpi];
      uyi=nv*[del(1),-del(3),del(4),-del(6)]';
      uzi=nw*[del(2),del(5)]';
      ipnel=ipx*(nel-1)+ip;
      Yp(ipnel)=nw*[Y(ijk(1,nel)),Y(ijk(2,nel))]';
      Zp(ipnel)=nw*[Z(ijk(1,nel)),Z(ijk(2,nel))]';
      uY(ipnel)= uyi*cji+uzi*sji;
      uZ(ipnel)=-uyi*sji+uzi*cji;
   end
end
axialstress;
strainenergy';
bar(strainenergy)
xlabel('beam elements and flexible joints')
ylabel('strain energy')
title('strain energy distribution')
% plot the deformed configuration
%mag=input('magnification factor of the displacement = ');
mag=0.05;
Zup=Zp+mag*uZ;
Yup=Yp+mag*uY;
plot(Zp,Yp,Zup,Yup)
title('deformed configuration')
xlabel('Z')
ylabel('Y')
%
force=-1000;
deflection=disp(8,2);
result(diter,1)=diter;
result(diter,2)=z4;
result(diter,3)=t;
result(diter,4)=force/deflection;
%
end
%
```

```
result
```

result =

1.0000	1.0000	0.8974	7.1342
2.0000	10.0000	0.9231	7.5908
3.0000	20.0000	0.9487	8.1027
4.0000	30.0000	0.9701	8.6340
5.0000	40.0000	0.9864	9.2560
6.0000	50.0000	0.9965	10.1074
7.0000	60.0000	1.0000	11.2818
8.0000	70.0000	0.9965	11.9089
9.0000	79.0000	0.9877	9.8952



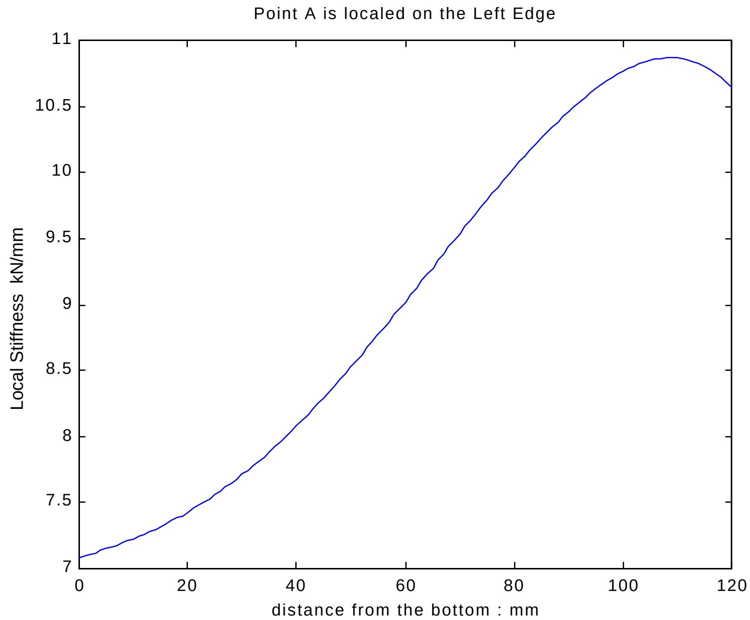
This graph yields that the maximum local stiffness can be obtained approximately at x = 68.5 mm from the left edge.

Case 2 : Point A is located along the left edge

result =

1.0000	1.0000	0.9004	7.0947
2.0000	10.0000	0.9577	7.2210
3.0000	20.0000	1.0290	7.4304
4.0000	30.0000	1.1094	7.7162
5.0000	40.0000	1.2000	8.0809
6.0000	50.0000	1.3016	8.5202
7.0000	60.0000	1.4142	9.0182

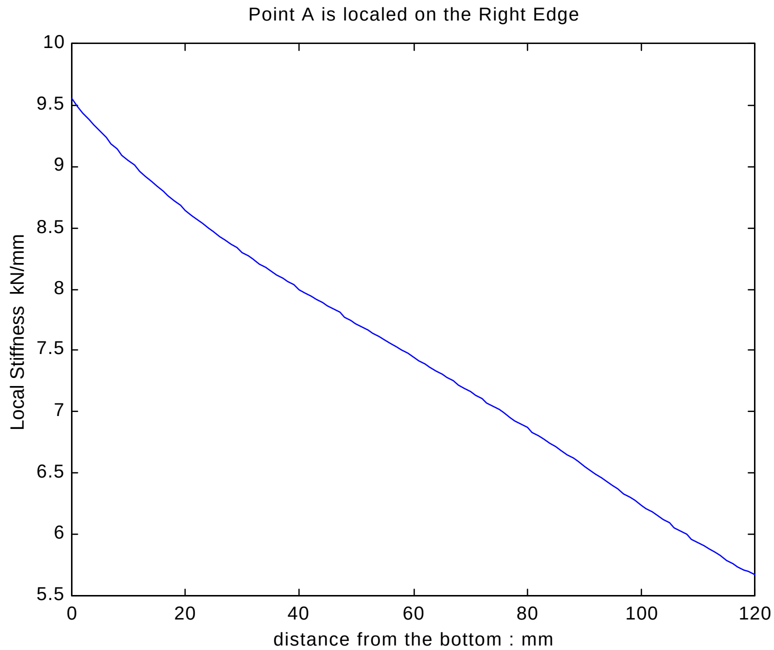
8.0000	70.0000	1.5364	9.5422
9.0000	80.0000	1.6641	10.0425
10.0000	90.0000	1.7889	10.4610
11.0000	100.0000	1.8974	10.7484
12.0000	110.0000	1.9728	10.8813
13.0000	119.0000	1.9997	10.6833



Case 3 : Point A is located along the right edge

result =

1.0000	1.0000	0.9945	9.4913
2.0000	10.0000	1.0733	9.0480
3.0000	20.0000	1.1767	8.6446
4.0000	30.0000	1.3016	8.3026
5.0000	40.0000	1.4552	7.9998
6.0000	50.0000	1.6483	7.7190
7.0000	60.0000	1.8974	7.4455
8.0000	70.0000	2.2283	7.1652
9.0000	80.0000	2.6833	6.8684
10.0000	90.0000	3.3282	6.5561
11.0000	100.0000	4.2426	6.2409
12.0000	110.0000	5.3666	5.9399
13.0000	119.0000	5.9925	5.6910


Point A is localed on the Right Edge
10
9.5
9
8.5
8
7.5
7
6.5
6
5.5
Local Stiffness kN/mm
0
20
40
60
80
100
120
distance from the bottom : mm