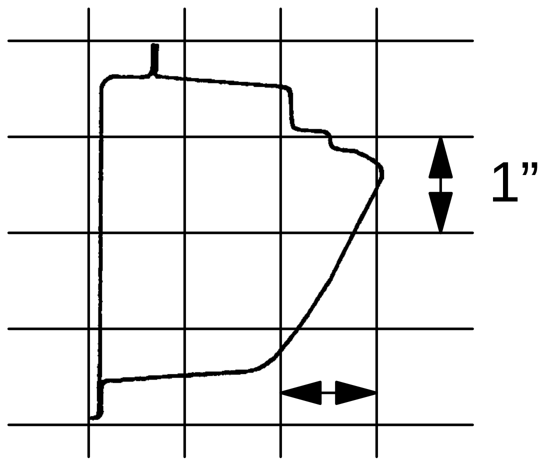
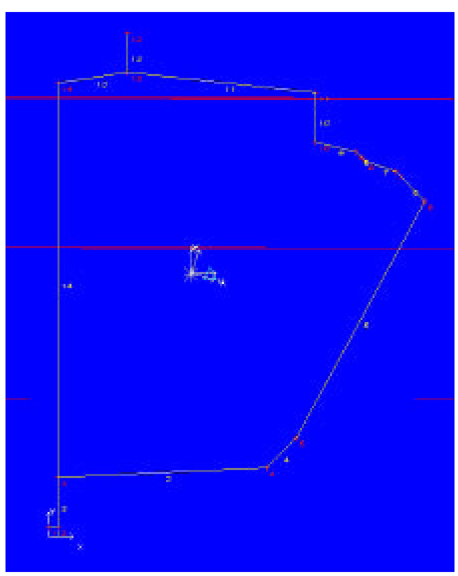
Homework #2
MEAM 599-02   Automotive Body Structures

1998 Winter

1. Using AISI/CARS’96_GAS or equivalent software such as UNIGRAPHICS if you are familiar with, find the geometric property of the following four cross sections. To do this, the thickness of the sheet metal to form the frames is assumed to be 1 mm. If a specific material characterization is required, you may pick up it from the database in AISI/CARS’96 for structural steel commonly used in automotive body. Since the geometry is given in approximately, you will determine the input to AISI/CARS’96 based on your judgement for details.

(a)





GAS - CARS Geometric Analysis of Sections
Version 5.0

Date: Apr 22 1998   Time: 16:35:04
Units : N, mm, MPa
Database : D:\CARS96\USER\HW2A
Section Name: Section A
Description :

Cross Section Geometry:

Point No.	X	Y
1	0	2
2	2	2
3	2	12
4	44	14
5	50	20
6	76	68
7	70	74
8	64	76
9	62	78
10	54	80
11	54	90
12	16	94
13	16	102
14	2	92

Line No.	Start Pt.	End Pt.	Length	Thickness	Material Archive	Line No.	Type
1	1	2	2	1	asdm.	1	Segment
2	2	3	10	2	asdm.	1	Segment
3	3	4	42.0476	1	asdm.	1	Segment
4	4	5	8.48528	1	asdm.	1	Segment
5	5	6	54.5894	1	asdm.	1	Segment
6	6	7	8.48528	1	asdm.	1	Segment
7	7	8	6.32456	1	asdm.	1	Segment
8	8	9	2.82843	1	asdm.	1	Segment
9	9	10	8.24621	1	asdm.	1	Segment
10	10	11	10	1	asdm.	1	Segment
11	11	12	38.2099	1	asdm.	1	Segment
12	12	13	8	2	asdm.	1	Segment
13	12	14	14.1421	1	asdm.	1	Segment
14	14	3	80	1	asdm.	1	Segment

Material Description:

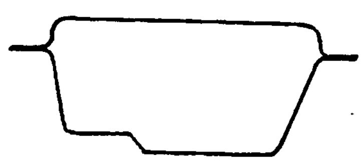
Archive	Mat'l No.	E	Fy	Archive Location
asdm.	1	203000	234.422	d:\cars96\

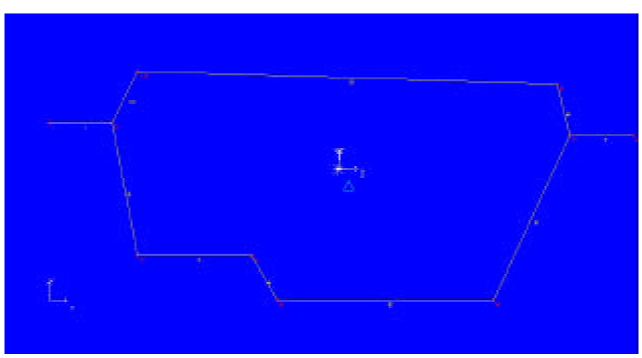
*** Results ***

Nominal Properties:

Area = 311.36
cx = 28.801
cy = 53.287
Ixx = 3.2029E+05
Iyy = 2.078E+05
Ixy = 40064
Sx+ = 6575
Sy+ = 4362
Sx- = -6184.8
Sy- = -7215.1
Theta = -17.731
Iuu = 3.331E+05
Ivv = 1.9499E+05
Su+ = 7722.6
Sv+ = 4759.6
Su- = -5733.4
Sv- = -5159.3
rx = 32.073
ry = 25.834
J = 3.189E+05
Cw = 0
ex = 32.5
ey = 53.336
Cuu = 2.8026
Cvv = 2.1508
Jopen = 48.667
Jc = 3.1885E+05
tomax = 2
tcmin = 1
Ao = 4668

(b)





GAS - CARS Geometric Analysis of Sections
Version 5.0

Date: Apr 22 1998   Time: 16:47:11
Units : N, mm, MPa
Database : D:\CARS96\USER\HW2A
Section Name: Section B
Description :

Cross Section Geometry:

Point No.	X	Y
1	0	28
2	10	28
3	14	7
4	32	7
5	36	0
6	70	0
7	82	26
8	92	26
9	80	34
10	14	36

Line No.	Start Pt.	End Pt.	Length	Thickness	Material Archive	Material No.	Line Type
1	1	2	10	2	asdm.	1	Segment
2	2	3	21.3776	1	asdm.	1	Segment
3	3	4	18	1	asdm.	1	Segment
4	4	5	8.06226	1	asdm.	1	Segment
5	5	6	34	1	asdm.	1	Segment
6	6	7	28.6356	1	asdm.	1	Segment
7	7	8	10	2	asdm.	1	Segment
8	7	9	8.24621	1	asdm.	1	Segment
9	9	10	66.0303	1	asdm.	1	Segment
10	10	2	8.94427	1	asdm.	1	Segment

Material Description:

Archive	Mat'l No.	E	Fy	Archive Location
asdm.	1	203000	234.422	d:\cars96\

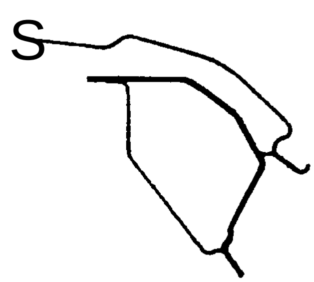
*** Results ***

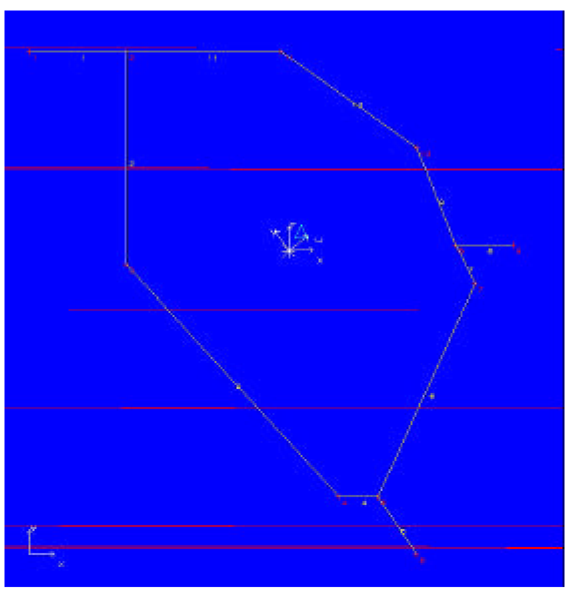
Nominal Properties:

Area = 233.3
cx = 45.614
cy = 20.683
Ixx = 41804
Iyy = 1.7901E+05
Ixy = -3466.9

Sx+	=	2643
Sy+	=	3859.2
Sx-	=	-1973.5
Sy-	=	-3924.5
Theta	=	-1.4465
Iuu	=	41716
Ivv	=	1.791E+05
Su+	=	2778.5
Sv+	=	3871.4
Su-	=	-1947.6
Sv-	=	-3909.7
rx	=	13.386
ry	=	27.701
J	=	94115
Cw	=	0
ex	=	47.163
ey	=	18.106
Cuu	=	1.8753
Cvv	=	3.9716
Jopen	=	53.333
Jc	=	94061
tomax	=	2
tcmin	=	1
Ao	=	2132

(c)





GAS - CARS Geometric Analysis of Sections
Version 5.0

Date: Apr 22 1998   Time: 16:54:24
Units   : N, mm, MPa
Database   : D:\CARS96\USER\HW2A
Section Name: Section C
Description :

Cross Section Geometry:

Point No.	X	Y
1	0	52
2	10	52
3	10	30
4	32	6
5	36	6
6	40	0
7	46	28
8	44	32
9	50	32
10	40	42
11	26	52

Line No.	Start Pt.	End Pt.	Length	Thickness	Material Archive	No.	Line Type
1	1	2	10	2	asdm.	1	Segment
2	2	3	22	1	asdm.	1	Segment
3	3	4	32.5576	1	asdm.	1	Segment
4	4	5	4	1	asdm.	1	Segment
5	5	6	7.2111	2	asdm.	1	Segment
6	5	7	24.1661	1	asdm.	1	Segment
7	7	8	4.47214	1	asdm.	1	Segment
8	8	9	6	2	asdm.	1	Segment
9	8	10	10.7703	1	asdm.	1	Segment
10	10	11	17.2047	1	asdm.	1	Segment
11	11	2	16	1	asdm.	1	Segment

Material Description:

Archive	Mat'l No.	E	Fy	Archive Location
asdm.	1	203000	234.422	d:\cars96\

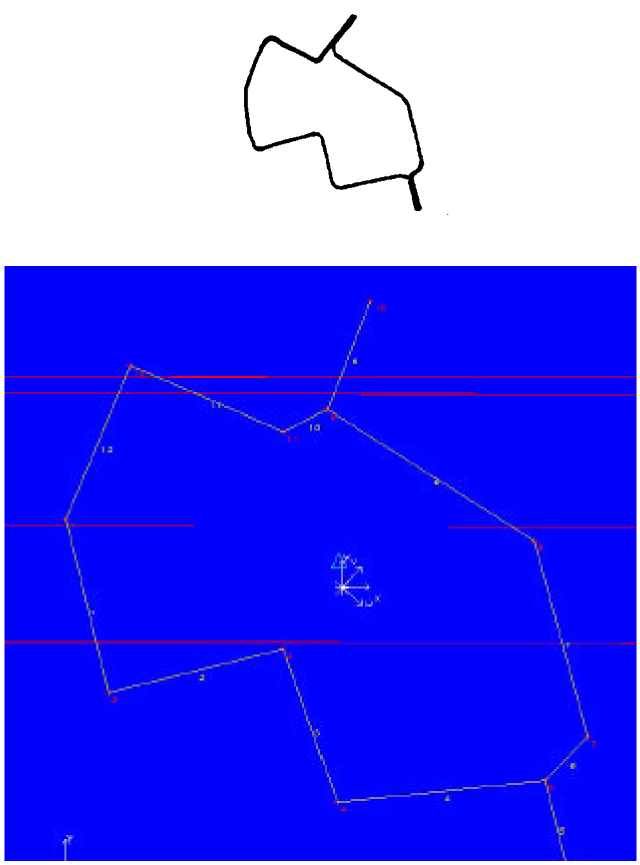
*** Results ***

Nominal Properties:

Area =   177.59
cx   =   26.757
cy   =   31.327

Ixx	=	50635
Iyy	=	36855
Ixy	=	-21803
Sx+	=	2336.3
Sy+	=	1585.7
Sx-	=	-1588.2
Sy-	=	-1377.4
Theta	=	36.231
Iuu	=	66610
Ivv	=	20880
Su+	=	2000.5
Sv+	=	1057.9
Su-	=	-2009.9
Sv-	=	-1410.9
rx	=	16.885
ry	=	14.406
J	=	37499
Cw	=	0
ex	=	28.054
ey	=	33.36
Cuu	=	3.6849
Cvv	=	2.0309
Jopen	=	61.896
Jc	=	37437
tomax	=	2
tcmin	=	1
Ao	=	1108

(d)



GAS - CARS Geometric Analysis of Sections
Version 5.0

Date: Apr 22 1998 Time: 17:00:46
Units : N, mm, MPa
Database : D:\CARS96\USER\HW2A
Section Name: Section D
Description :

Cross Section Geometry:

Point No.	X	Y
1	0	32
2	4	16
3	20	20
4	25	6
5	44	8
6	46	0
7	48	12
8	43	30
9	24	42
10	28	52
11	20	40
12	6	46

Line No.	Start Pt.	End Pt.	Length	Thickness	Material Archive	Material No.	Line Type
1	1	2	16.4924	1	asdm.	1	Segment
2	2	3	16.4924	1	asdm.	1	Segment
3	3	4	14.8661	1	asdm.	1	Segment
4	4	5	19.105	1	asdm.	1	Segment
5	5	6	8.24621	2	asdm.	1	Segment
6	5	7	5.65685	1	asdm.	1	Segment
7	7	8	18.6815	1	asdm.	1	Segment
8	8	9	22.4722	1	asdm.	1	Segment
9	9	10	10.7703	2	asdm.	1	Segment
10	9	11	4.47214	1	asdm.	1	Segment
11	11	12	15.2315	1	asdm.	1	Segment
12	12	1	15.2315	1	asdm.	1	Segment

Material Description:

Archive	Mat'l No.	E	Fy	Archive Location
asdm.	1	203000	234.422	d:\cars96\

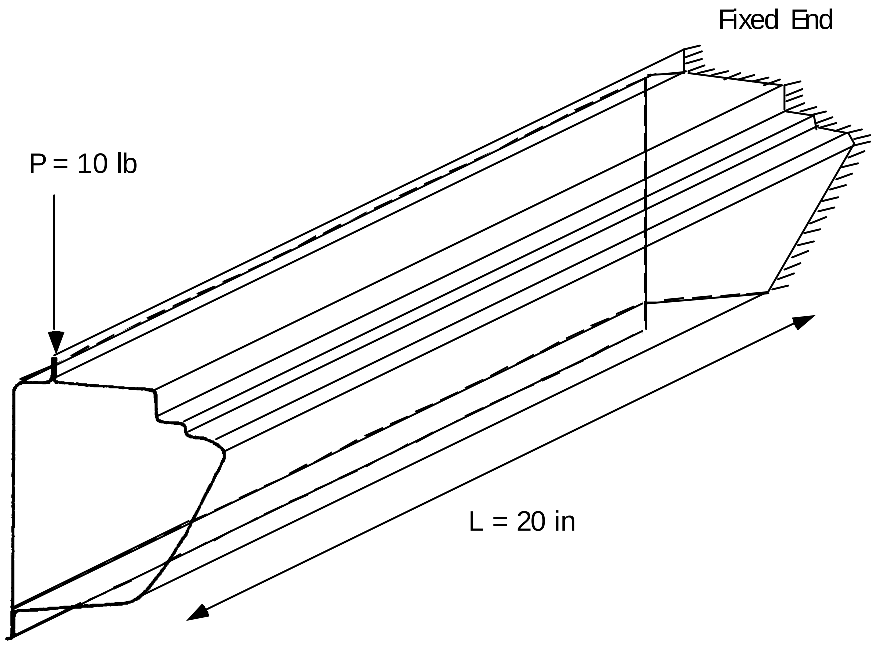
*** Results ***

Nominal Properties:

Area = 186.73
cx = 25.34

cy = 25.642
Ixx = 42109
Iyy = 43555
Ixy = -17376
Sx+ = 1575.4
Sy+ = 1882.1
Sx- = -1626.8
Sy- = -1686.5
Theta = -43.809
Iuu = 25441
Ivv = 60223
Su+ = 1197.9
Sv+ = 1814.4
Su- = -1148.3
Sv- = -2111.8
rx = 15.017
ry = 15.272
J = 36685
Cw = 0
ex = 25.057
ey = 28.127
Cuu = 2.2934
Cvv = 1.7485
Jopen = 50.711
Jc = 36634
tomax = 2
tcmin = 1
Ao = 1167

2. Assuming the cross section given in 1-(a), we consider a cantilever beam shown in the following figure.



Although this cantilever should be regarded as a short beam, assuming that this is modeled as a beam we have studied so far,

(a) Find the axial stress distribution on the cross section at the fixed end of the cantilever, and

(b) Find the horizontal and vertical deflections as well as the angle of twist of the beam at the free end.

To do so, you may model this cantilever by a beam element. Since the one end is fixed, you will end up a 6 x 6 matrix equation that can be solved by MATLAB. Unknowns are the two transverse deflections of the shear center, the average axial displacement, three rotations about the *x*, y, and *z* axes, respectively, where *x* and *y* are the principal axes of the cross section. For your analysis, you should specify what material constants are applied for structural steel. Here, we expect that Young's modulus is about 200 Gpa (30 M psi), and Poisson's ratio is about 0.29.

It is also noted that the vertical load *P* is applied not on the shear center, it generates twist moment about the shear center. Thus, you will have three input, say, V_x, V_y, and M_z.

(a) From the result in 1-(a) by CARS96 we have

$$I_{uu} = 3.331 \times 10^5 \ mm^4 \quad , \quad I_{vv} = 1.95 \times 10^5 \ mm^4 \quad , \quad J_c = 3.2 \times 10^5 \ mm^4$$

and the principal axes is rotated clockwise 17.7 degree. Thus the vertically applied force P generates the two components of the transverse loads in the principal axis directions P_u

and P_v as well as the toque about the axis through the shear center $M_z = P\, e_x$, where ex is the perpendicular distance of the shear center to the line of vertical force obtained as e_x = 32.5mm-16mm :

$$P_u = P \sin\theta = 13.5240\ N$$
$$P_v = -P \cos\theta = -42.3763\ N$$
$$M_z = Pe_x = 733.9530\ N.mm$$

The transverse loads Pu and Pv generates the bending moments at the fixed end, and they are

$$M_u = P_v L = -2.1527 \times 10^4\ \ N.mm$$
$$M_v = P_u L = 6.8702 \times 10^3\ \ N.mm$$

These generate the axial stress

$$\sigma_z = -\frac{M_u}{I_{uu}} v - \frac{M_u}{I_{vv}} u$$

where (u,v) are the coordinates of a point of the cross section from the centroid of the cross section in the u and v principal directions, respectively. Thus, using the following MATLAB program, we can compute the axial stress due to the transverse load P. Here it is noted that torque M_z does not contribute the axial stress.

```
% MATLAB Program for HW #2 - 2 (a)
% 599-02   Automotive Body Structures
%_____________________________________________
% define the reference coordinate of the nodes
xr=[0,2,2,44,50,76,70,64,62,54,54,16,16,2];
yr=[2,2,12,14,20,68,74,76,78,80,90,94,102,92];
% define the line segments
ijk=[1,2;2,3;3,4;4,5;5,6;6,7;7,8;8,9;9,10;10,11;11,12;12,13;12,14;14,3];
% draw the cross section
[nelx,node]=size(ijk)
for nel=1:nelx
    xp(1)=xr(ijk(nel,1));
    xp(2)=xr(ijk(nel,2));
    yp(1)=yr(ijk(nel,1));
    yp(2)=yr(ijk(nel,2));
    plot(xp,yp)
    if nel==1, hold, end
end
hold
% location of the centroid
cx=28.8;
cy=53;
% location of the shear center
```

```
sx=32.5;
sy=53.3;
% distance between the shear center and loading line
ex=32.5-16;
% angle of the principal axes
theta=17.7*pi/180;
% applied load  P = 10 lb = 44.482 N
P=44.482;
% moment due to the applied load not on the shear center
Mz=P*ex
% components of the applied force P in the principal directions
Pu=P*sin(theta)
Pv=-P*cos(theta)
% moment of inertia with respect to the principal axes
Iuu=3.331*10^5;
Ivv=1.95*10^5;
Jc=3.2*10^5;
% length of the cantilever
L=508;
% bending moments by the applied load
Mu=Pv*L
Mv=Pu*L
% axial stress due to the bending
xs=(xr-cx)*cos(theta)-(yr-cy)*sin(theta)
ys=(yr-cy)*cos(theta)+(xr-cx)*sin(theta)
sz=-(Mu/Iuu)*ys-(Mv/Ivv)*xs
% plot the stress distribution
pause
for nel=1:nelx
    xp(1)=xr(ijk(nel,1));
    xp(2)=xr(ijk(nel,2));
    yp(1)=yr(ijk(nel,1));
    yp(2)=yr(ijk(nel,2));
    zp0(1)=0;
    zp0(2)=0;
    zp1(1)=sz(ijk(nel,1));
    zp1(2)=sz(ijk(nel,2));
    plot3(xp,yp,zp0,xp,yp,zp1)
    if nel==1, hold, end
end
hold
```

```
nelx =

    14


node =

     2
```

Mz =

733.9530

Pu =

13.5240

Pv =

-42.3763

Mu =

-2.1527e+004

Mv =

6.8702e+003

xs =

Columns 1 through 7

-15.3646 -13.4593 -16.4996   22.9041   26.7959   36.9715   29.4313

Columns 8 through 14

23.1073   20.5939   12.3645   9.3242 -28.0931 -30.5253 -40.8223

ys =

Columns 1 through 7

-58.7526 -58.1445 -48.6179 -33.9432 -26.4030   27.2296   31.1213

Columns 8 through 14

31.2025   32.4997   31.9728   41.4994   33.7568   41.3781   27.5950

sz =

Columns 1 through 7

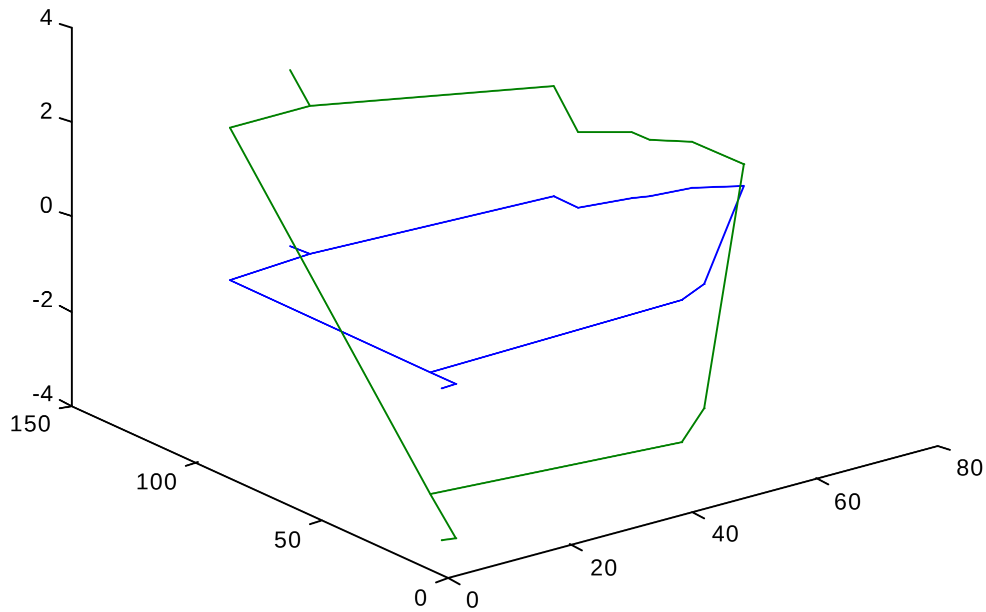
-3.2557   -3.2835   -2.5607   -3.0006   -2.6504   0.4572   0.9744

Columns 8 through 14

1.2024 1.3748 1.6307 2.3535 3.1714 3.7496 3.2216

Current plot held
Current plot released

Therefore, the maximum stress is 3.7596 N/mm2 at node 13, and the minimum stress is – 3.2835 N/mm2 at node 2, and the distribution can be obtained as follows :



Check : Under the assumption that the cross section is almost rectangular with the moment of inertia Ixx = 3.2029×10^5 mm^4, while the vertical load P = 10 lb is the dominant force, we shall obtain the maximum and minimum stress due to the bending moment M = PL = 44.482 x 508 Nmm :

$$\sigma_+ = \frac{M}{I_{xx}} y_+ = \frac{44.482 \times 508}{3.2029 \times 10^5} \times 48.7 = 3.436 N/mm^2$$

$$\sigma_- = -\frac{M}{I_{xx}} y_- = -\frac{44.482 \times 508}{3.2029 \times 10^5} \times 52.3 = 3.690 N/mm^2$$

Therefore, the value we have by MATLAB can be regarded as the one we expect.

(b) Using the stiffness matrix of the 3 dimensional beam element, we can calculate the deflections as follows by using the MATLAB program :

```
% MATLAB Program : Homework 2 - 2(b)
%_______________________________________________________
% input data for the stiffness matrix
E=200000;
v=0.29;
G=E/(2*(1+v));
Fx=0;
Fy=0;
Ixx=3.331*10^5;
Iyy=1.9499*10^5;
Jc=3.189*10^5;
A=311.36;
L=508;
% element stiffness matrix for 3D beam element
ske=zeros(12);
ske(1,1)=12*E*Ixx/((1+Fx)*L^3);
ske(5,1)=ske(1,1)*L/2;
ske(7,1)=-ske(1,1);
ske(11,1)=ske(5,1);
ske(2,2)=12*E*Iyy/((1+Fy)*L^3);
ske(4,2)=-ske(2,2)*L/2;
ske(8,2)=-ske(2,2);
ske(10,2)=ske(4,2);
ske(3,3)=E*A/L;
ske(9,3)=-ske(3,3);
ske(4,4)=(4+Fy)*E*Iyy/((1+Fy)*L);
ske(8,4)=-ske(4,2);
ske(10,2)=(2-Fy)*E*Iyy/((1+Fy)*L);
ske(5,5)=(4+Fx)*E*Ixx/((1+Fx)*L);
ske(7,5)=-ske(5,1);
ske(11,5)=(2-Fx)*E*Ixx/((1+Fx)*L);
ske(6,6)=G*Jc/L;
ske(12,6)=-ske(6,6);
ske(7,7)=ske(1,1);
ske(11,7)=-ske(5,1);
ske(8,8)=ske(2,2);
ske(10,8)=-ske(4,2);
ske(9,9)=ske(3,3);
ske(10,10)=ske(4,4);
ske(11,11)=ske(5,5);
ske(12,12)=ske(6,6);
for i=1:11
    for j=i+1:12
        ske(i,j)=ske(j,i);
    end
end
```

```
ske;
% reduced stiffness matrix : second node is fixed
sker=ske(1:6,1:6)
fe=zeros(6,1);
% applied transverse loads and torque
fe(1)=-42.3763;
fe(2)=13.5240;
fe(6)=733.9530;
fe
% result of the transverse deflections etc
ue=inv(sker)*fe


sker =

  1.0e+008 *

    0.0001         0         0         0    0.0155         0
         0    0.0000         0   -0.0091         0         0
         0         0    0.0012         0         0         0
         0   -0.0091         0    3.0707         0         0
    0.0155         0         0         0    5.2457         0
         0         0         0         0         0    0.4866


fe =

  -42.3763
   13.5240
         0
         0
         0
  733.9530


ue =

   -0.0278
    0.0152
         0
    0.0000
    0.0001
    0.0000
```

Noting that these values are for the principal axes, we must convert them into the horizontal and vertical components of the displacement :

Horizontal Displacement +0.0060
Vertical Displacement   -0.0311

Check : Again, using the simple beam theory, the vertical deflection can be approximated by

$$d = -\frac{PL^3}{3EI} = -\frac{44.482 \times 508^3}{3 \times 200000 \times 3.203 \times 10^5} = -0.0303mm$$