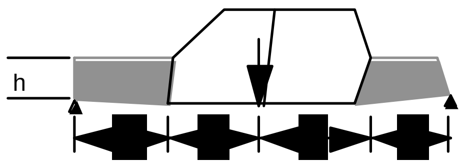
Homework 3

For the side frame on the following sheet

A. Compute the vehicle bending deflection Δ



F=1500 Lb.
a = 20 in.
b = 40
c = 40
d = 10
h = 30

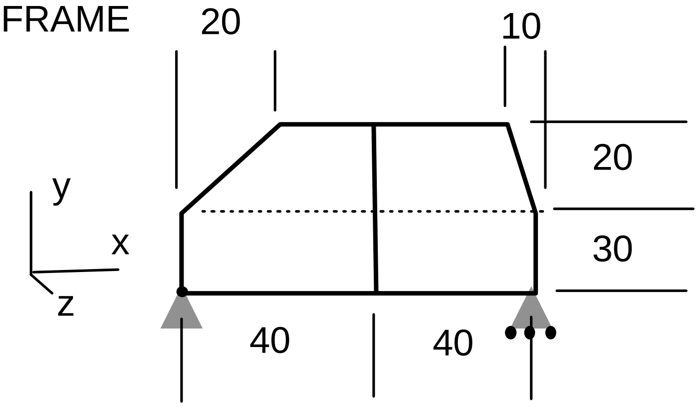
- Dimensions in Inches
- Frame is planar
- All metal thickness are .040 in
- Material Mild Steel
- Neglect Flanges - All sections are closed
- Sections are normal to beam axis
- All joints rigid except as noted

B. If the value for Δ computed in (A) exceeds the deflection requirement, which beam would you alter first and why?

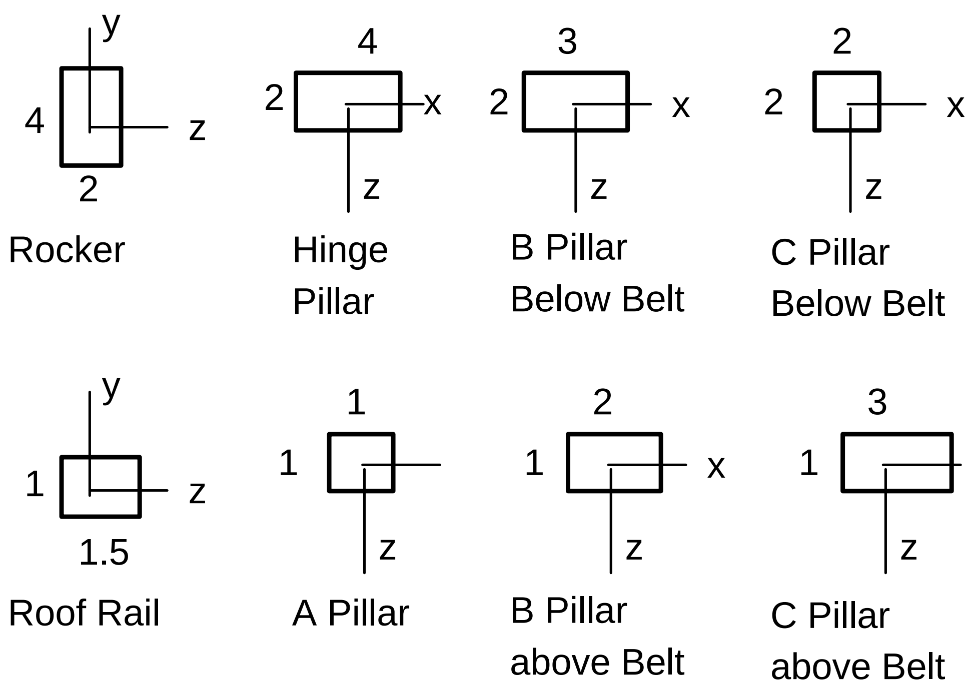
C. Compute $(Gt)_{EFF}$ for torsion.
Take Q=1650Lb

D. If the value for $(Gt)_{EFF}$ computed in (C) is too low compared to the requirement, which beam would you alter first and why?

SIDE FRAME

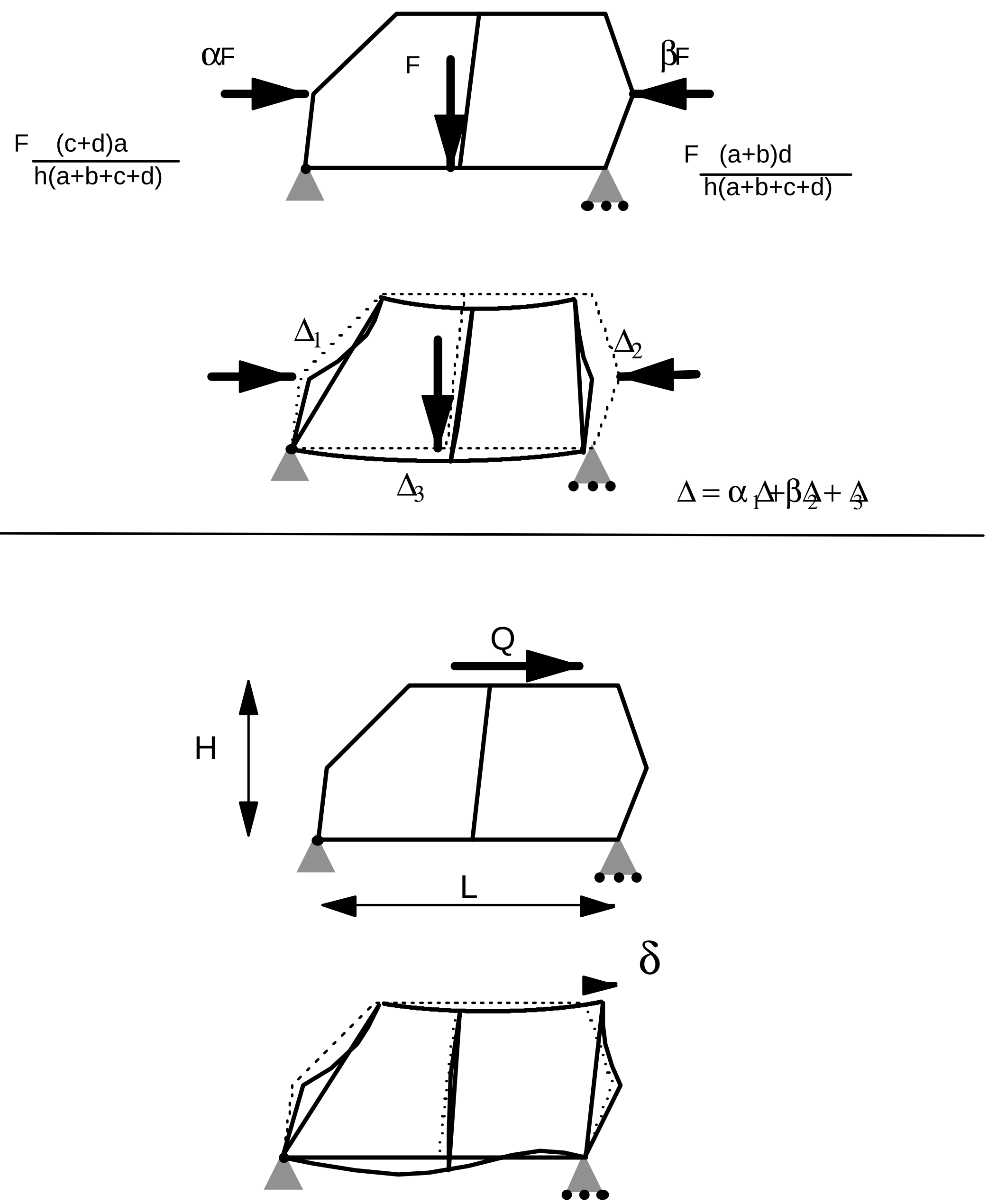


SECTIONS



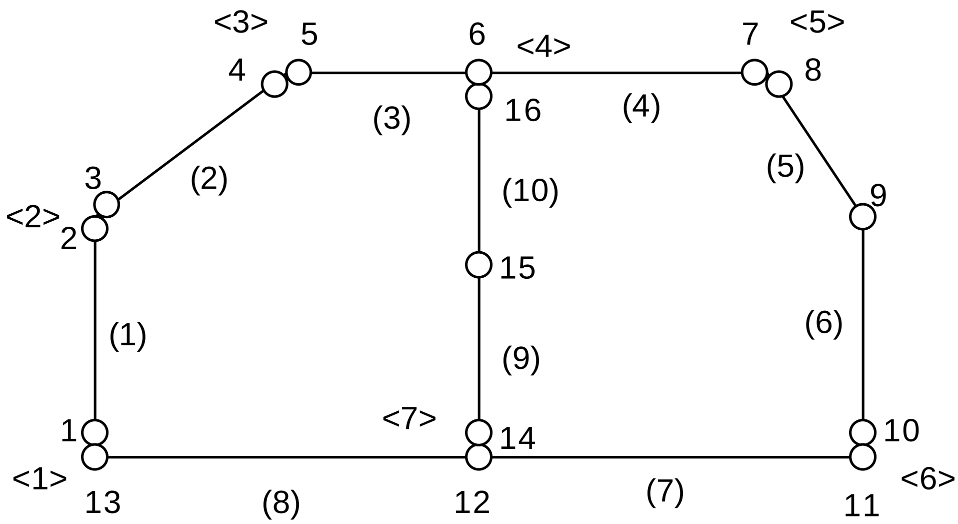
JOINT RATES Kzz (Nm/rad)

A Pillar-Hinge Pillar Hinge Pillar - Rocker B Pillar-Rocker C Pillar- Rocker	.2E6
All Connections to Roof Rail	.01E6



$$(Gt)_{EFF} = (Q/\delta)(L/H)$$

(B)




```
% MATLAB Program : hw3_98W
%
% MEAM 599- 02 / 1998 Winter
% FEM for Side Frame Analysis using plane beam elements
% with possibly flexible joints
%
% is modified for Homework #3
%
%
__________________________________________________________________________________________________
_____________________
%
% Pre-Processing / set up an analysis model
%
% read nodal coordinates ( Z, Y )
nx=16;
Z=[0,0,0,20,20,40,70,70,80,80,80,40,0,40,40,40];
Y=[0,30,30,50,50,50,50,50,30,0,0,0,0,0,30,50];
% plot the nodes of the plane beam structure
plot(Z,Y,'+')
% read element connectivity and section type of beam elements
%     ijk(1,nel)=node i of beam element nel
%     ijk(2,nel)=node j of beam element nel
%     ijk(3,nel)=section type of beam element nel
nelx=10;
ijk=[1,2,2;
   3,4,6;
   5,6,5;
   6,7,5;
   8,9,8;
   9,10,4;
```

```
   12,11,1;
   13,12,1;
   14,15,3;
   15,16,7]';
% plot the beam elements
for nel=1:nelx
      Ze(2*nel-1)=Z(ijk(1,nel));
      Ze(2*nel)=Z(ijk(2,nel));
      Ye(2*nel-1)=Y(ijk(1,nel));
      Ye(2*nel)=Y(ijk(2,nel));
end
plot(Z,Y,'+',Ze,Ye)
% read section type ( properties )
nsecx=8;
Ei=30000000;
t=0.04;
Iyy1=(2*4^3-(2-2*t)*(4-2*t)^3)/12;
Iyy2=(2*4^3-(2-2*t)*(4-2*t)^3)/12;
Iyy3=(2*3^3-(2-2*t)*(3-2*t)^3)/12;
Iyy4=(2*2^3-(2-2*t)*(2-2*t)^3)/12;
Iyy5=(1.5*1^3-(1.5-2*t)*(1-2*t)^3)/12;
Iyy6=(1*1^3-(1-2*t)*(1-2*t)^3)/12;
Iyy7=(1*2^3-(1-2*t)*(2-2*t)^3)/12;
Iyy8=(1*3^3-(1-2*t)*(3-2*t)^3)/12;
E=Ei*[1,1,1,1,1,1,1,1];
A=[0.48,0.48,0.4,0.32,0.2,0.16,0.24,0.32];
Iyy=[Iyy1,Iyy2,Iyy3,Iyy4,Iyy5,Iyy6,Iyy7,Iyy8];
Fy=[0,0,0,0,0,0,0,0];
% read data for flexible joints
nfjx=7;
fjoint=[1,13;12,14;11,10;2,3;4,5;16,6;8,7]';
kZ=10^8*[1,1,1,1,1,1,1];
kY=kZ;
kq1=0.2*10^6;
kq2=0.01*10^6;
kqX=8.851*[kq1,kq1,kq1,kq1,kq2,kq2,kq2];
% plot the flexible joints
if nfjx>0
      for i=1:nfjx
            Zfj(i)=Z(fjoint(1,i));
            Yfj(i)=Y(fjoint(1,i));
      end
plot(Z,Y,Ze,Ye,'+',Zfj,Yfj,'o')
end
% displacement constraints
spc=[];
```

```
nspc=3;
spc=[13,1,0;13,2,0;11,1,0]';
% applied forces and moments at the nodes
afm=[];
nafm=3;
alfa=10/33;
beta=6/33;
afm=[2,2,alfa*1500;9,2,-beta*1500;12,1,-1500]';
% afm=[5,2,412.5;6,2,825;7,2,412.5]';
%
%
% FE-Processing / forming the global stiffness matrix
%
% beam elements
%
sk=zeros(3*nx);
f=zeros(3*nx,1);
sksize=3*nx;
%
for nel=1:nelx
%
    Zji=Z(ijk(2,nel))-Z(ijk(1,nel));
    Yji=Y(ijk(2,nel))-Y(ijk(1,nel));
    Lji=sqrt(Zji^2+Yji^2);
    cji=Zji/Lji;
    sji=Yji/Lji;
    TG=zeros(6);
    TG(1,1)= cji;
    TG(1,2)=-sji;
    TG(2,1)= sji;
    TG(2,2)= cji;
    TG(3,3)= 1;
    TG(4,4)= cji;
    TG(4,5)=-sji;
    TG(5,4)= sji;
    TG(5,5)= cji;
    TG(6,6)= 1;
%
    skel=zeros(6);
    ijk3=ijk(3,nel);
    EIyy=E(ijk3)*Iyy(ijk3);
    Fy1=(1+Fy(ijk3));
    skel(1,1)=12*E(ijk3)*Iyy(ijk3)/((1+Fy(ijk3))*Lji^3);
    skel(2,2)=E(ijk3)*A(ijk3)/Lji;
    skel(3,1)=-6*E(ijk3)*Iyy(ijk3)/((1+Fy(ijk3))*Lji^2);
    skel(3,3)=(4+Fy(ijk3))*E(ijk3)*Iyy(ijk3)/((1+Fy(ijk3))*Lji);
```

```
    skel(4,1)=-skel(1,1);
    skel(4,3)=-skel(3,1);
    skel(4,4)= skel(1,1);
    skel(5,2)=-skel(2,2);
    skel(5,5)= skel(2,2);
    skel(6,1)= skel(3,1);
    skel(6,3)=(2-Fy(ijk3))*E(ijk3)*Iyy(ijk3)/((1+Fy(ijk3))*Lji);
    skel(6,4)=-skel(3,1);
    skel(6,6)= skel(3,3);
    for i=1:5
        for j=1+1:6
            skel(i,j)=skel(j,i);
        end
    end
%
    skeg=TG'*skel*TG;
%
    ijk1=ijk(1,nel);
    ijk2=ijk(2,nel);
    ndg=[3*ijk1-2,3*ijk1-1,3*ijk1,3*ijk2-2,3*ijk2-1,3*ijk2];
    for i=1:6
        for j=1:6
            sk(ndg(i),ndg(j))=sk(ndg(i),ndg(j))+skeg(i,j);
        end
    end
%
end
%
% flexible joint elements
%
if nfjx>0
    for nfj=1:nfjx
        skeg=zeros(6);
        ijk1=fjoint(1,nfj);
        ijk2=fjoint(2,nfj);
        kYb=(sk(3*ijk1-2,3*ijk1-2)+sk(3*ijk2-2,3*ijk2-2))/2;
        kZb=(sk(3*ijk1-1,3*ijk1-1)+sk(3*ijk2-1,3*ijk2-1))/2;
        kqXb=(sk(3*ijk1,3*ijk1)+sk(3*ijk2,3*ijk2))/2;
    refrencestiffness=[nfj,kYb,kZb,kqXb];
    kYb=1;
    kZb=1;
    kqXb=1;
        skeg(1,1)= kY(nfj)*kYb;
        skeg(4,4)= kY(nfj)*kYb;
        skeg(1,4)=-kY(nfj)*kYb;
        skeg(4,1)=-kY(nfj)*kYb;
```

```
        skeg(2,2)= kZ(nfj)*kZb;
        skeg(5,5)= kZ(nfj)*kZb;
        skeg(2,5)=-kZ(nfj)*kZb;
        skeg(5,2)=-kZ(nfj)*kZb;
        skeg(3,3)= kqX(nfj)*kqXb;
        skeg(6,6)= kqX(nfj)*kqXb;
        skeg(3,6)=-kqX(nfj)*kqXb;
        skeg(6,3)=-kqX(nfj)*kqXb;
        ndg=[3*ijk1-2,3*ijk1-1,3*ijk1,3*ijk2-2,3*ijk2-1,3*ijk2];
        for i=1:6
            for j=1:6

    sk(ndg(i),ndg(j))=sk(ndg(i),ndg(j))+skeg(i,j);
            end
        end
    end
end
%
% single point constraints
%
for i=1:nspc
    spc1=spc(1,i);
    spc2=spc(2,i);
    spc3=spc(3,i);
    ndof=3*(spc1-1)+spc2;
    f=f-sk(:,ndof)*spc3;
    sk(:,ndof)=zeros(sksize,1);
    sk(ndof,:)=zeros(1,sksize);
    sk(ndof,ndof)=1;
end
%
% applied for#es and moments
%
for i=1:nafm
    afm1=afm(1,i);
    afm2=afm(2,i);
    afm3=afm(3,i);
    ndof=3*(afm1-1)+afm2;
    f(ndof)=f(ndof)+afm3;
end
%
% solving the matrix equation
%
d=sk\f;
disp=[];
for i=1:nx
```

```
   disp(i,1)=i;
   disp(i,2)=d(3*i-2);
   disp(i,3)=d(3*i-1);
   disp(i,4)=d(3*i);
end
disp
%
% Post-Processing of the computed results
%
strainenergy=[];
axialstress=[];
Zp=[];
Yp=[];
uZ=[];
uY=[];
for nel=1:nelx
    Zji=Z(ijk(2,nel))-Z(ijk(1,nel));
    Yji=Y(ijk(2,nel))-Y(ijk(1,nel));
    Lji=sqrt(Zji^2+Yji^2);
    cji=Zji/Lji;
    sji=Yji/Lji;
    TG=zeros(6);
    TG(1,1)= cji;
    TG(1,2)=-sji;
    TG(2,1)= sji;
    TG(2,2)= cji;
    TG(3,3)= 1;
    TG(4,4)= cji;
    TG(4,5)=-sji;
    TG(5,4)= sji;
    TG(5,5)= cji;
    TG(6,6)= 1;
    deg=zeros(6,1);
    for i=1:2
        ijki=ijk(i,nel);
        for j=1:3
            deg(3*(i-1)+j)=d(3*(ijki-1)+j);
        end
    end
    del=TG*deg;
    skel=zeros(6);
    ijk3=ijk(3,nel);
    EIyy=E(ijk3)*Iyy(ijk3);
    Fy1=(1+Fy(ijk3));
    skel(1,1)=12*E(ijk3)*Iyy(ijk3)/((1+Fy(ijk3))*Lji^3);
    skel(2,2)=E(ijk3)*A(ijk3)/Lji;
```

```
    skel(3,1)=-6*E(ijk3)*Iyy(ijk3)/((1+Fy(ijk3))*Lji^2);
    skel(3,3)=(4+Fy(ijk3))*E(ijk3)*Iyy(ijk3)/((1+Fy(ijk3))*Lji);
    skel(4,1)=-skel(1,1);
    skel(4,3)=-skel(3,1);
    skel(4,4)= skel(1,1);
    skel(5,2)=-skel(2,2);
    skel(5,5)= skel(2,2);
    skel(6,1)= skel(3,1);
    skel(6,3)=(2-Fy(ijk3))*E(ijk3)*Iyy(ijk3)/((1+Fy(ijk3))*Lji);
    skel(6,4)=-skel(3,1);
    skel(6,6)= skel(3,3);
    for i=1:5
        for j=1+1:6
            skel(i,j)=skel(j,i);
        end
    end
    nel;
    strainenergynel=(del'*skel*del)/2;
    strainenergy(nel)=strainenergynel;
% axial strain  ez = ez0 + y ( ezi*(1-z/l) + ezj*(z/l) )
    ez0=(del(5)-del(2))/Lji;
    ezi= 6*(del(1)-del(4))/Lji^2-(4*del(3)+2*del(6))/Lji;
    ezj=-6*(del(1)-del(4))/Lji^2+(2*del(3)+4*del(6))/Lji;
    axialstrain=[ez0,ezi,ezj];
% axial stress  sz = sz0 + y ( szi*(1-z/l) + szj*(z/l) )
    sz0=E(ijk3)*ez0;
    szi=E(ijk3)*ezi;
    szj=E(ijk3)*ezj;
  axialstress(nel,1)=sz0;
  axialstress(nel,2)=szi;
  axialstress(nel,3)=szj;
% displacement of the beam axis
    ipx=11;
    for ip=1:ipx
        zpi=(ip-1)/(ipx-1);
        nv=[1-3*zpi^2+2*zpi^3,Lji*(zpi-2*zpi^2+zpi^3),3*zpi^2-2*zpi^3,Lji*(-
zpi^2+zpi^3)];
        nw=[1-zpi,zpi];
        uyi=nv*[del(1),-del(3),del(4),-del(6)]';
        uzi=nw*[del(2),del(5)]';
        ipnel=ipx*(nel-1)+ip;
        Yp(ipnel)=nw*[Y(ijk(1,nel)),Y(ijk(2,nel))]';
        Zp(ipnel)=nw*[Z(ijk(1,nel)),Z(ijk(2,nel))]';
        uY(ipnel)= uyi*cji+uzi*sji;
        uZ(ipnel)=-uyi*sji+uzi*cji;
    end
```

```
end
axialstress
if nfjx>0
    for nfj=1:nfjx
        skeg=zeros(6);
        ijk1=fjoint(1,nfj);
        ijk2=fjoint(2,nfj);
        kYb=(sk(3*ijk1-2,3*ijk1-2)+sk(3*ijk2-2,3*ijk2-2))/2;
        kZb=(sk(3*ijk1-1,3*ijk1-1)+sk(3*ijk2-1,3*ijk2-1))/2;
    kqXb=(sk(3*ijk1,3*ijk1)+sk(3*ijk2,3*ijk2))/2;
    kYb=1;
    kZb=1;
    kqXb=1;
        skeg(1,1)= kY(nfj)*kYb;
        skeg(4,4)= kY(nfj)*kYb;
        skeg(1,4)=-kY(nfj)*kYb;
        skeg(4,1)=-kY(nfj)*kYb;
        skeg(2,2)= kZ(nfj)*kZb;
        skeg(5,5)= kZ(nfj)*kZb;
        skeg(2,5)=-kZ(nfj)*kZb;
        skeg(5,2)=-kZ(nfj)*kZb;
        skeg(3,3)= kqX(nfj)*kqXb;
        skeg(6,6)= kqX(nfj)*kqXb;
        skeg(3,6)=-kqX(nfj)*kqXb;
        skeg(6,3)=-kqX(nfj)*kqXb;
        for i=1:2
            ijki=fjoint(i,nfj);
            for j=1:3
                deg(3*(i-1)+j)=d(3*(ijki-1)+j);
            end
        end
        nfj;
        strainenergynfj=(deg'*skeg*deg)/2;
        strainenergy(nelx+nfj)=strainenergynfj;
    end
end
strainenergy'
bar(strainenergy)
xlabel('beam elements and flexible joints')
ylabel('strain energy')
title('strain energy distribution')
% plot the deformed configuration
mag=input('magnification factor of the displacement = ');
Zup=Zp+mag*uZ;
Yup=Yp+mag*uY;
plot(Zp,Yp,Zup,Yup)
```

```
title('deformed configuration')
xlabel('Z')
ylabel('Y')


disp =

    1.0000    0.0000    0.0000    0.0124
    2.0000   -0.0005    0.3646    0.0116
    3.0000   -0.0005    0.3646    0.0105
    4.0000    0.0855    0.2751    0.0032
    5.0000    0.0855    0.2751    0.0202
    6.0000   -0.3275    0.2738    0.0063
    7.0000    0.1132    0.2721   -0.0106
    8.0000    0.1132    0.2721    0.0111
    9.0000   -0.0005    0.0433    0.0100
   10.0000    0.0000    0.0005   -0.0084
   11.0000         0    0.0005   -0.0110
   12.0000   -0.3295    0.0003    0.0001
   13.0000         0         0    0.0123
   14.0000   -0.3295    0.0003    0.0018
   15.0000   -0.3285    0.1229    0.0057
   16.0000   -0.3275    0.2738    0.0082


axialstress =

  1.0e+004 *

   -0.0455    0.0137   -0.1820
   -0.2678   -7.9234    6.3659
   -0.1937    4.5792   -8.7258
   -0.1624   -9.2314    5.8434
   -0.0915    0.5626   -0.8569
   -0.0516   -1.4556   -2.2324
    0.0108   -2.1032    0.4358
    0.0239   -0.0137   -1.8147
    0.0959    0.5747    0.2106
    0.1599    0.8769   -0.1339


ans =

    0.5783
   20.1737
   21.1140
   36.0879
    2.5106
   34.6997
   84.5257
   75.8851
    4.4508
    1.1271
```

0.0059
2.4894
5.6799
0.9915
12.7879
0.1566
20.8227

magnification factor of the displacement = 10

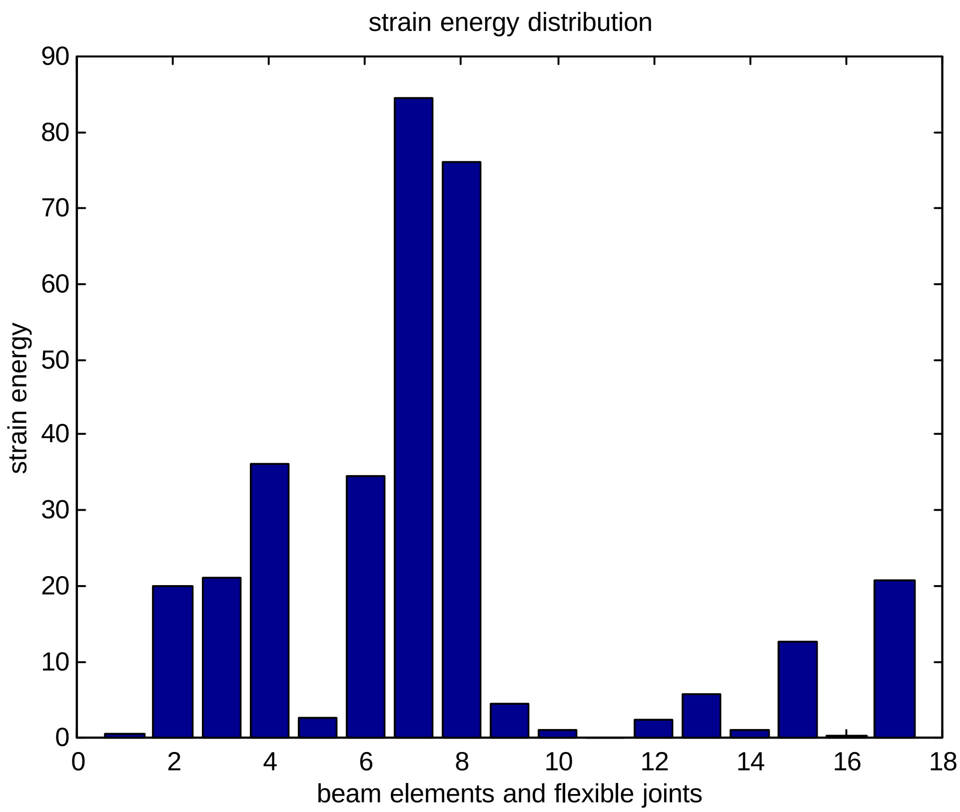
From the above calculation, we have

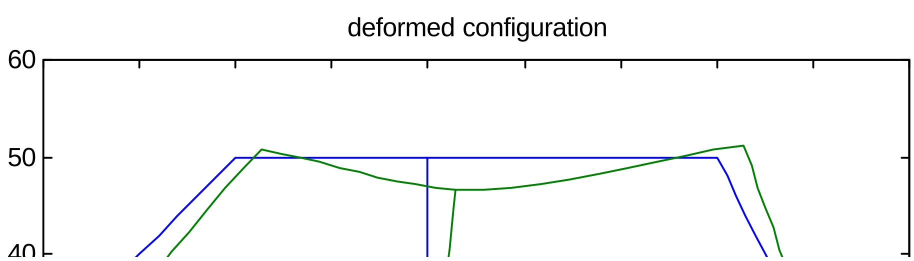
$$\mathrm{a} = \frac{(c+d)a}{h(a+b+c+d)} = \frac{10}{33} \quad , \quad \mathrm{b} = \frac{(a+b)d}{h(a+b+c+d)} = \frac{6}{33}$$

$$\Delta_1 = 0.3646in \quad , \quad \Delta_2 = -0.0433in \quad , \quad \Delta_3 = 0.3295in$$

and then

$$\Delta = \mathrm{a}\Delta_1 + \mathrm{b}\Delta_2 + \Delta_3 = \frac{10}{33}\times 0.364 - \frac{6}{33}\times 0.0423 + 0.3295 = 0.4321in$$





Check : Suppose that only the rocker can resist to the vertical force F. Then, using the simply supported beam theory, the vertical deflection at the loading point becomes

$$\Delta = \frac{\partial}{\partial P}\int_0^L \frac{M^2}{2EI}dx = \frac{1}{2EI}\frac{\partial}{\partial P}\left\{\int_0^a \left(P\frac{b}{L}x\right)^2 dx + \int_0^b \left(P\frac{a}{L}x\right)^2 dx\right\} = \frac{P}{EI}\frac{a^2b^2}{3L}$$

where a is the distance of the loading point from the left end point of the beam whose length is L = a + b, and b is the distance of the loading point from the right end point of the beam. If the point force F is applied at the center of the beam, we have

$$\Delta = \frac{PL^3}{48EI}$$

Rocker's moment of inertia I is computed by

$$I = \frac{1}{12}\left(2\times 4^3 - 1.92\times 3.92^3\right) = 1.0289 in^4$$

and then

$$\Delta = \frac{1500\times 80^3}{48\times 30000000\times 1.0289} = 0.5184 .$$

If we add the moment of inertia of the roof rail, then the moment of inertia becomes

$$I = 1.0289 + \frac{1}{12}\left(1.5\times1^3 - 1.42\times0.92^3\right) = 1.0618$$

Then the deflection becomes 0.5023. From this we can expect that Δ_3 is the same order of this but it must be smaller than this value.

(C)

disp =

1.0000	0.0000	0.0000	0.0257
2.0000	0.0008	0.9997	0.0377
3.0000	0.0008	0.9997	0.0390
4.0000	-0.9103	1.9170	0.0137
5.0000	-0.9103	1.9170	-0.0264
6.0000	-0.1725	1.9178	-0.0219
7.0000	0.4808	1.9179	0.0057
8.0000	0.4808	1.9179	0.0465
9.0000	-0.0006	0.9534	0.0477
10.0000	0.0000	0.0038	0.0058
11.0000	0	0.0038	-0.0023
12.0000	-0.1715	0.0027	0.0011
13.0000	0	0	0.0131
14.0000	-0.1715	0.0027	0.0144
15.0000	-0.1720	0.9488	0.0430
16.0000	-0.1725	1.9178	0.0434

axialstress =

1.0e+005 *

0.0079	0.2170	0.0222
0.0462	0.9651	-1.5012
0.0128	-1.0799	1.2144
0.0009	-0.5453	1.0980
-0.0106	0.1057	-0.0732
-0.0056	-0.1243	-0.7145
0.0082	-0.1915	0.1395
0.0205	-0.2170	0.0373
-0.0049	0.4558	0.1152
-0.0082	0.4796	-0.4661

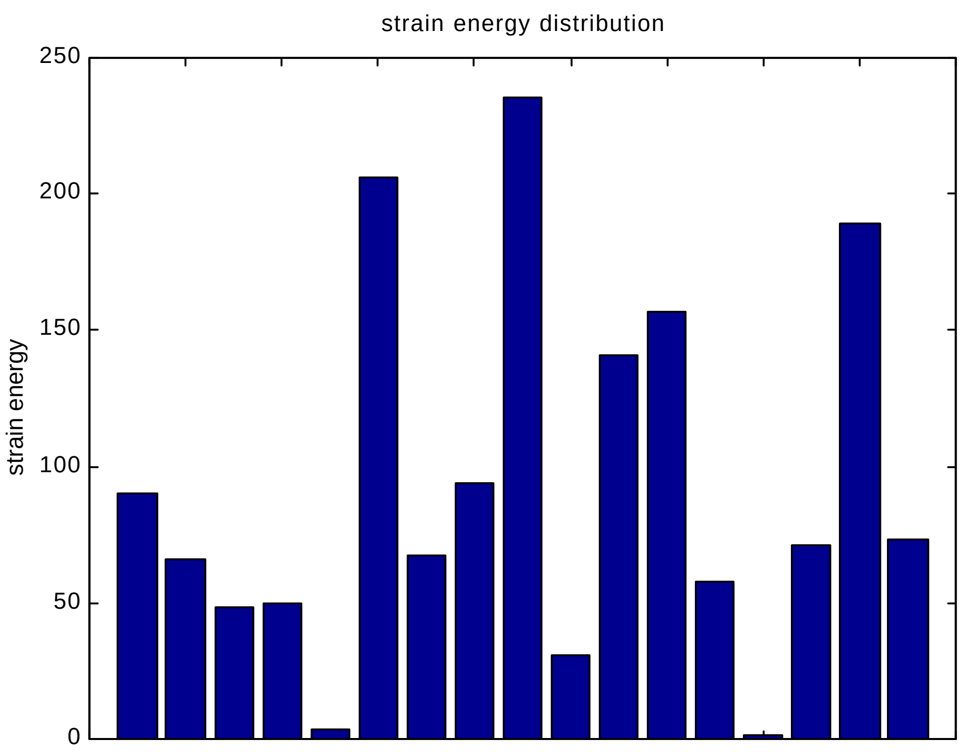
ans =

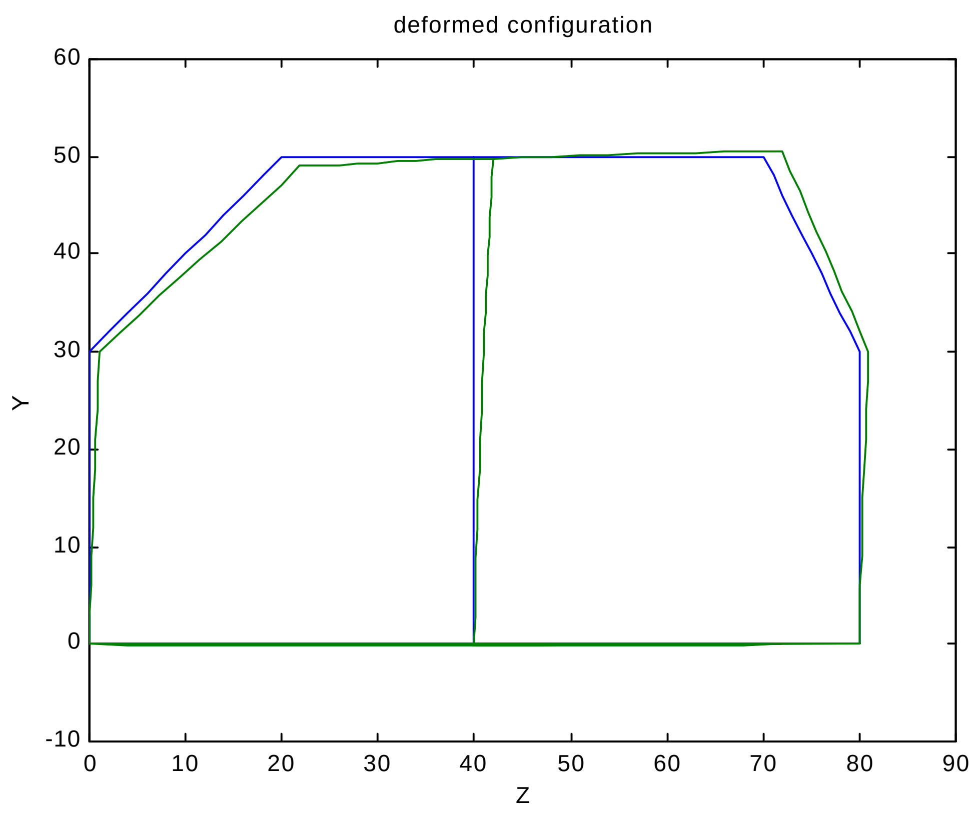
90.0255
66.0875
48.6455
49.5110
3.8608
205.8508
67.4600
93.7045
235.5112
30.8859
140.8595
156.5499
58.1791
1.4725
71.1119
188.8320
73.5134

magnification factor of the displacement = 1

In this case we have

$$\left(Gt\right)_{eff} = \frac{Q}{\delta}\frac{L}{H} = \frac{1650}{1.97170}\frac{80}{50} = 1377\ lb/in$$





Check : B-pillar moment of inertia is approximately obtained as

$$I = \frac{1}{12}\left(2\times 3^3 - 1.92\times 2.92^3\right) = 0.5165 in^4$$

and the deflection of this beam by the horizontal force P=Q/3=1650/3 becomes

$$\Delta_B = \frac{PL^3}{3EI} = \frac{1650\times 50^3}{9\times 30000000\times 0.5165} = 1.4790 in$$

under the assumption that the B-pillar is rigidly connected to the rocker beam. Since we have flexibility of the joint at the connection of the B-pillar and rocker beam, we have additional displacement :

$$\Delta_J = L\theta = \frac{PL^2}{k_\theta} = \frac{1650\times 50^2}{3\times 8.851\times 10^6} = 0.7767 in$$

that is, the total horizontal displacement can be estimated by

$$\Delta = \Delta_B + \Delta_J = 2.2557 in.$$