Topology Optimization

Mathematics for Design

Homogenization Design Method
(HMD)

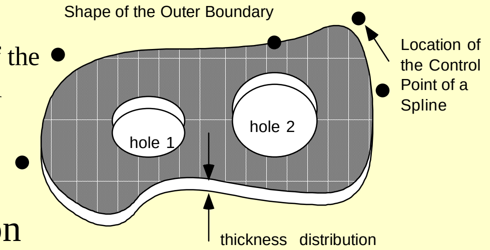
Why topology ?

Change in shape & size may not lead our design criterion for reduction of structural weight.

Structural Design

3 Sets of Problems

- Sizing Optimization
  - thickness of a plate or membrane
  - height, width, radius of the cross section of a beam
- Shape Optimization
  - outer/inner shape
- Topology Optimization
  - number of holes
  - configuration



Sizing Optimization

Starting of Design Optimization

1950s : Fully Stressed Design

$$\bar{\sigma} = \sigma_{allowable} \quad \textit{in a structure}$$

1960s : Mathematical Programming (L. Schmit at UCLA)

$$\min_{\substack{\bar{\sigma} \le \sigma_{allowable} \\ \|\boldsymbol{u}\| \le u_{\max}}} \quad \textit{Total Weight}$$

➡ Design Sensitivity Analysis

Equilibrium : State Equation

$$\mathbf{Ku} = \mathbf{f}$$

Design Sensitivity

$$\frac{Dg}{D\mathbf{d}} = \frac{\partial g}{\partial \mathbf{d}} + \underbrace{\frac{\partial \mathbf{u}}{\partial \mathbf{d}}}_{\textit{Design Velocity Sensitivity}} \frac{\partial g}{\partial \mathbf{u}}$$

Performance Functions g

$$\mathbf{Ku} = \mathbf{f} \quad \Rightarrow \quad \frac{\partial \mathbf{K}}{\partial \mathbf{d}}\mathbf{u} + \mathbf{K}\frac{\partial \mathbf{u}}{\partial \mathbf{d}} = \frac{\partial \mathbf{f}}{\partial \mathbf{d}}$$

$$\boxed{\frac{Dg}{D\mathbf{d}} = \frac{\partial g}{\partial \mathbf{d}} + \mathbf{K}^{-1}\left(-\frac{\partial \mathbf{K}}{\partial \mathbf{d}}\mathbf{u} + \frac{\partial \mathbf{f}}{\partial \mathbf{d}}\right)\frac{\partial g}{\partial \mathbf{u}}}$$

Typical Performance Functions

Strain Energy Density
For Structural Design (This must be constant !)

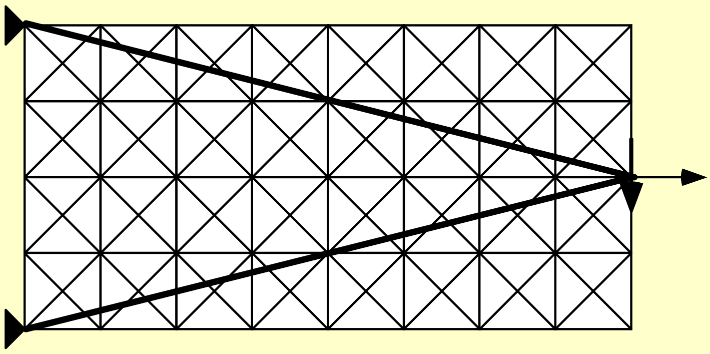
Mises Equivalent Stress
For Strength Design and Failure Analysis

Mean Compliance & Maximum Displacement
For Stiffness Design

Maximum Strain
For Formability Study of Sheet Metals

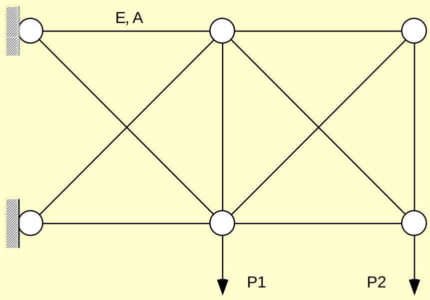
Hemp in 1950s

Size to Topology



Eliminate unnecessary bars by designing the cross sectional area.

An Optimization Algorithm



$$\min_{\substack{\boldsymbol{Ku}=\boldsymbol{f} \\ \sigma_e \le \sigma_{allowable} \\ \|\boldsymbol{u}_i\| \le u_{\max}}} \sum_{e=1}^{N_{\max}} \rho_e A_e L_e$$

Design Sensitivity

$$\boldsymbol{K}\frac{\partial \boldsymbol{u}}{\partial A_e} = -\frac{\partial \boldsymbol{K}}{\partial A_e}\boldsymbol{u} + \frac{\partial \boldsymbol{f}}{\partial A_e}$$

$$\frac{\partial \sigma_{\hat{e}}}{\partial A_e} = \frac{\partial \left(\boldsymbol{D}_{\hat{e}} \boldsymbol{B}_{\hat{e}} \boldsymbol{u}_{\hat{e}} \right)}{\partial A_e} = \boldsymbol{D}_{\hat{e}} \boldsymbol{B}_{\hat{e}} \frac{\partial \boldsymbol{u}_{\hat{e}}}{\partial A_e}$$

$$\frac{\partial \|\boldsymbol{u}_i\|}{\partial A_e} = \frac{\boldsymbol{u}_i}{\|\boldsymbol{u}_i\|} \bullet \frac{\partial \boldsymbol{u}_i}{\partial A_e}$$

Prager in 1960s

Design Optimization Theory

Maximizing the minimum total potential energy

$$\Pi = \sum_{e=1}^{N_e} \Pi_e = \sum_{e=1}^{N_e} \frac{1}{2}\mathbf{d}_e^{\,T}\mathbf{K}_e\mathbf{d}_e - \mathbf{d}_e^{\,T}\mathbf{f}_e$$

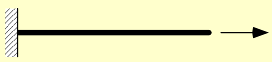
$$\max_{\substack{design \\ A_e}} \; \underbrace{\min_{\mathbf{d}_e} \Pi}_{\text{Leads Equilibrium}}$$

Why Total Potential ?

Maximizing the Global Stiffness

Minimizing the mean compliance (Prager)
when forces are applied

$$\min_{design} \; \mathbf{u}^T\mathbf{f}_s$$



Maximizing the mean compliance
when displacement is specified

$$\max_{design} \; \mathbf{u}_s^{\,T}\mathbf{f}$$

Lagrangian

$$L = \underbrace{\frac{1}{2}\sum_{e=1}^{N_E} \mathbf{d}_e^{\,T}\mathbf{K}_e\mathbf{d}_e - \mathbf{d}_e^{\,T}\mathbf{f}_e}_{\text{Total Potential Energy}} + \lambda\underbrace{\left(\sum_{e=1}^{N_E}\rho_e A_e L_e - W\right)}_{\text{Weight Constraint}}$$

Variation

$$\delta L = \sum_{e=1}^{N_E} \delta\mathbf{d}_e^{\,T}\left(\mathbf{K}_e\mathbf{d}_e - \mathbf{f}_e\right) + \left(\frac{1}{2}\mathbf{d}_e^{\,T}\frac{\partial\mathbf{K}_e}{\partial A_e}\mathbf{d}_e - \lambda\rho_e L_e\right)\delta A_e$$

$$+\delta\lambda\left(\sum_{e=1}^{N_E}\rho_e A_e L_e - W\right)$$

Optimality Condition

$$\mathbf{K}_e\mathbf{d}_e = \mathbf{f}_e$$

$$\frac{1}{2}\mathbf{d}_e^{\,T}\frac{\partial\mathbf{K}_e}{\partial A_e}\mathbf{d}_e + \lambda\rho_e L_e = 0$$

Something must be Constant !

$$\sum_{e=1}^{N_E}\rho_e A_e L_e - W \le 0$$

$$\frac{1}{2}\mathbf{d}_e^{\,T}\frac{1}{\rho_e L_e}\frac{\partial\mathbf{K}_e}{\partial A_e}\mathbf{d}_e = -\lambda$$

Physical Meaning

Strain Energy Density Must be Constant

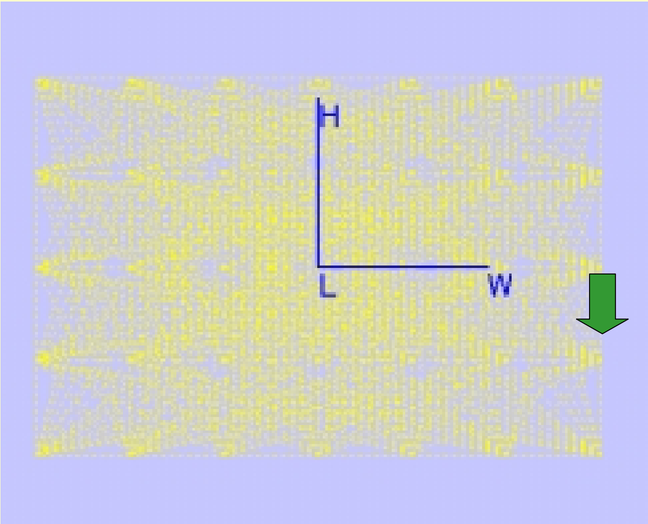
$$\frac{1}{2}\mathbf{d}_e^{\,T}\frac{1}{\rho_e L_e}\frac{\partial \mathbf{K}_e}{\partial A_e}\mathbf{d}_e = -\lambda$$

Prager's Condition

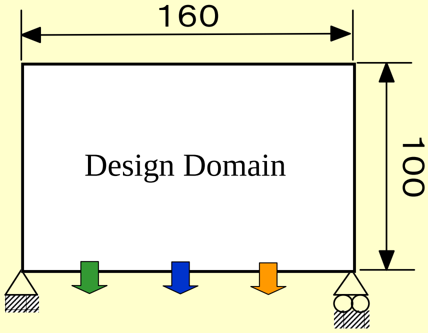
$$\Leftrightarrow$$

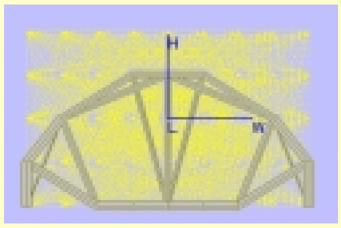
$$\frac{1}{2}\mathbf{d}_e^{\,T}\underbrace{\frac{1}{\rho_e A_e L_e}\mathbf{K}_e}_{\text{Weight Average of the Stiffness}}\mathbf{d}_e = -\lambda$$

Example 1

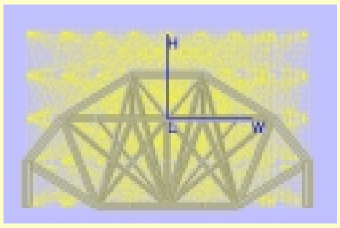


Example 2



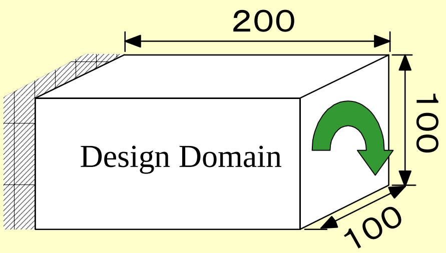


(a) Single Loading

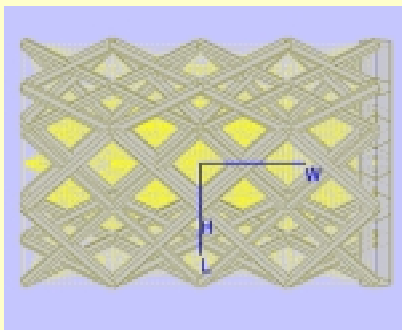


(b) Multiple Loading

Example 3



Applying Torque

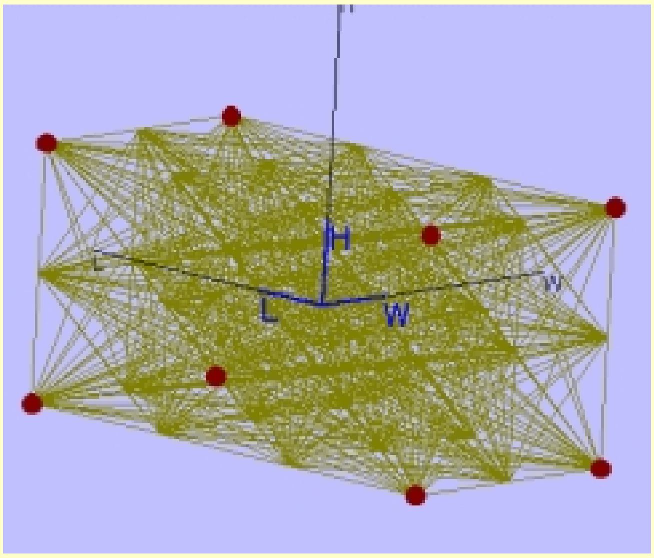


TOPODANUKI

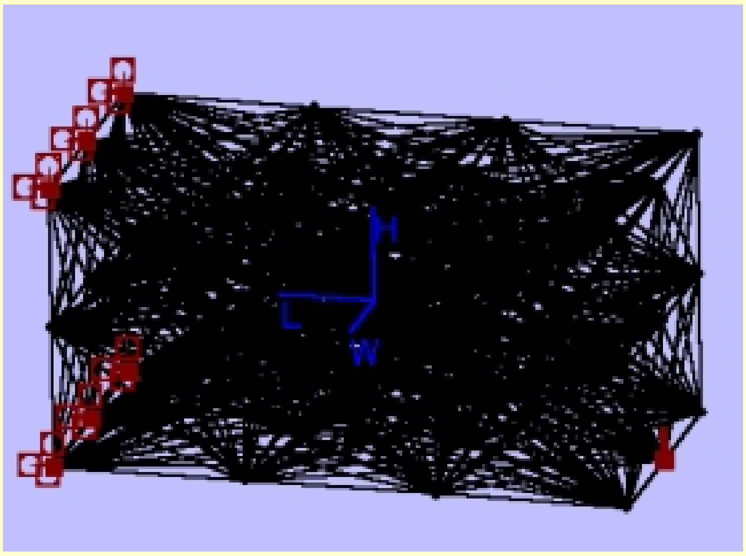
A Topology Optimization Soft

Toyota Central R&D Labs.

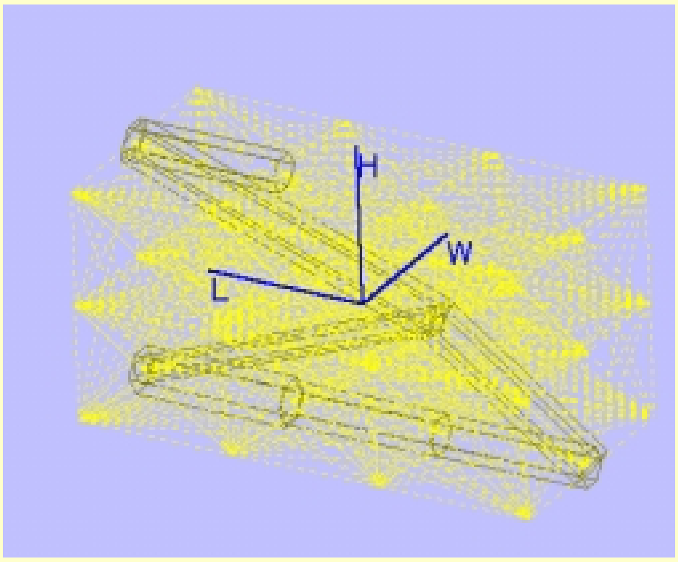
Making up a grand-structure



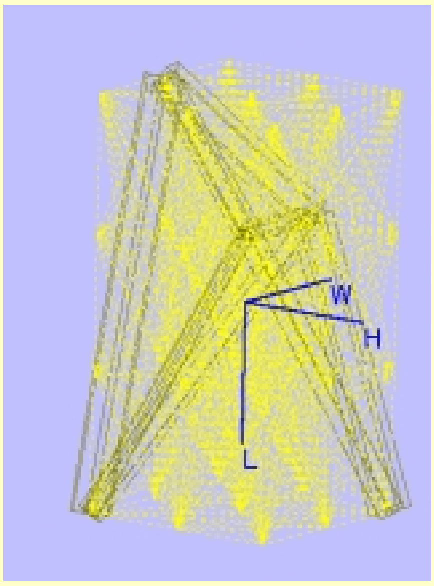
Set up support and load conditions



Only a bending load is applied

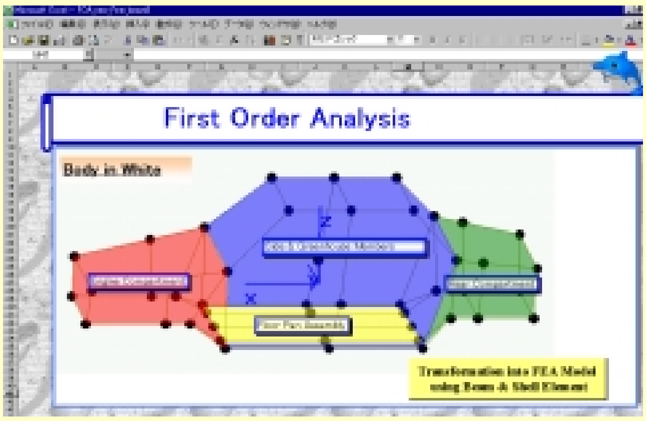


Two Loads are applied

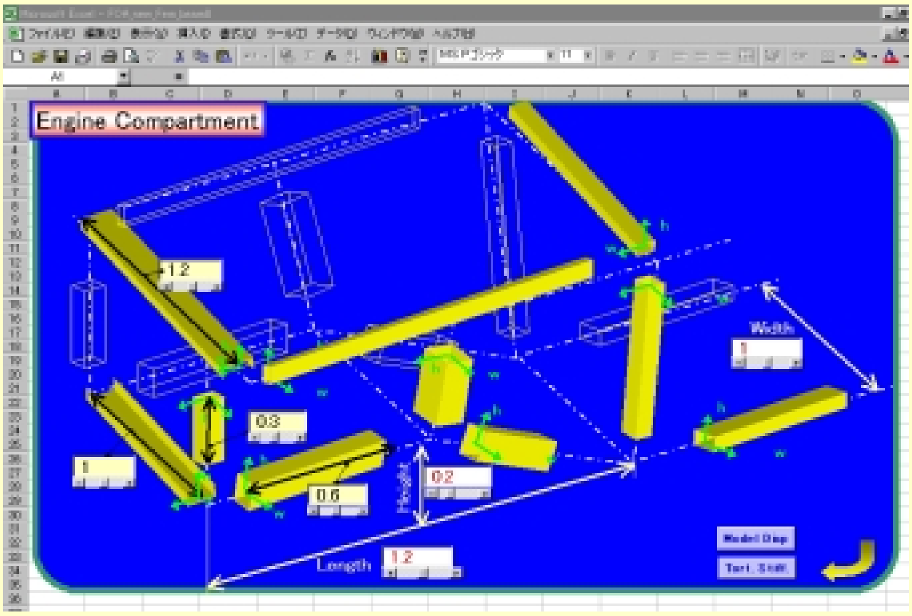


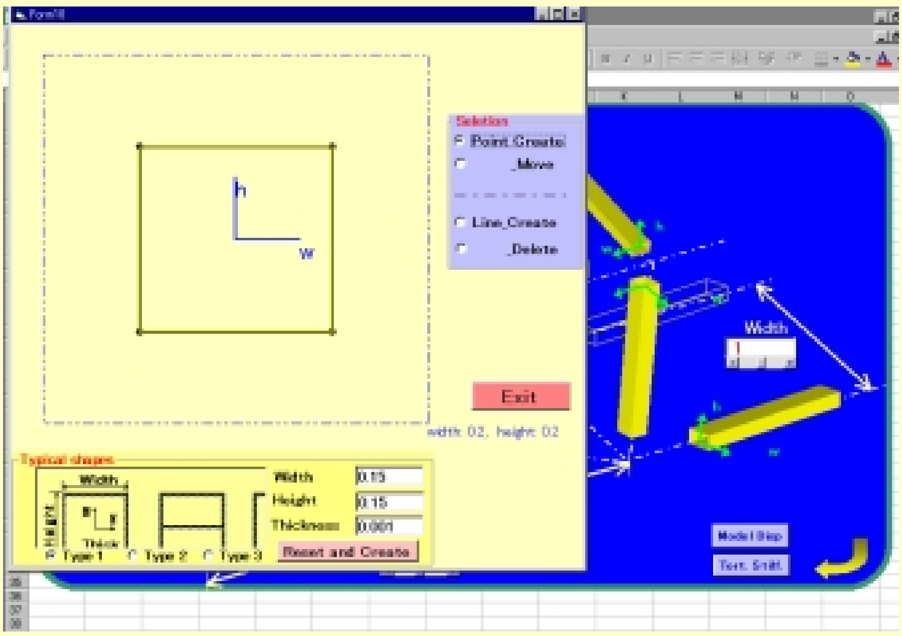
Further Development

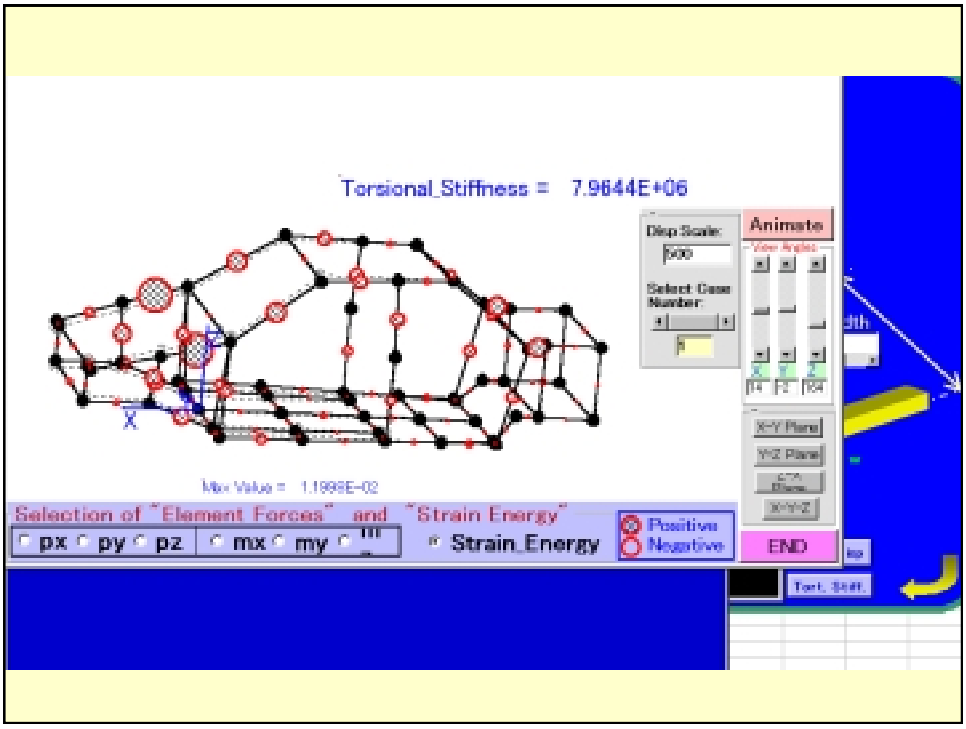
First Order Analysis in Toyota Central R&D

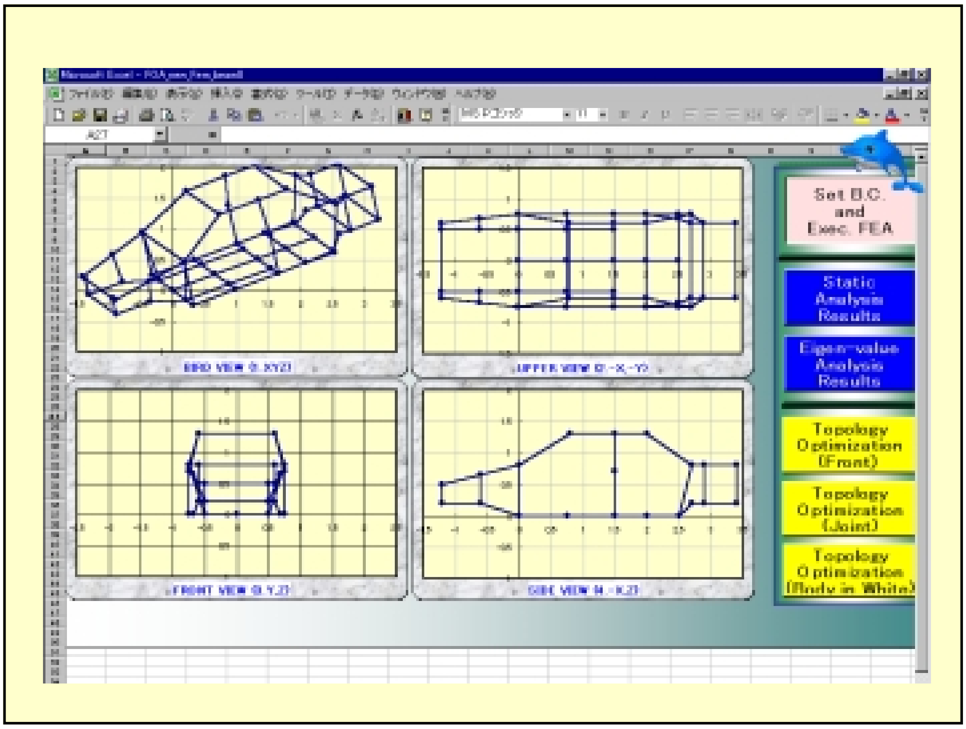


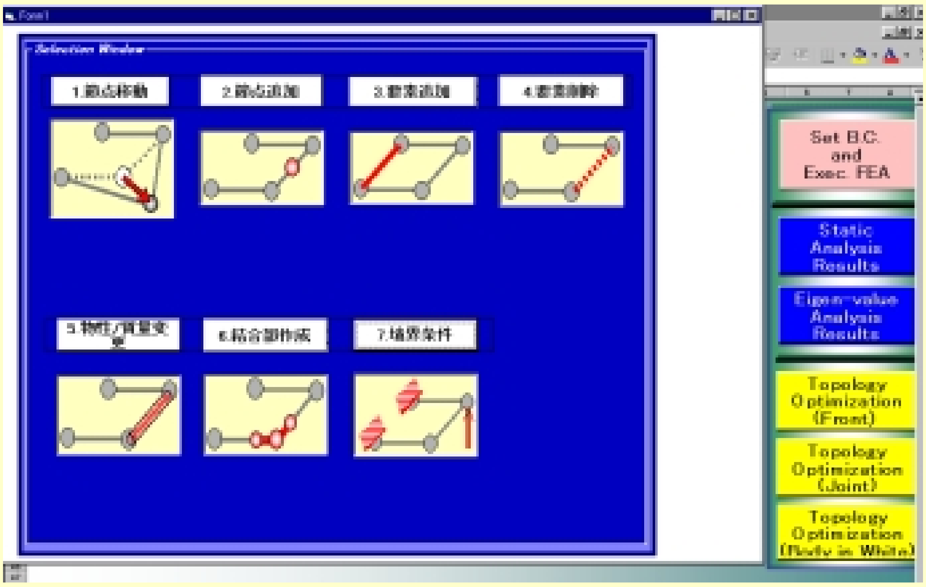
Microsoft EXCEL Based Software


Engine Compartment
1.2
Width
1
0.3
1
0.6
0.2
Height
Length
1.2
Model Disp
Tort. Stiff

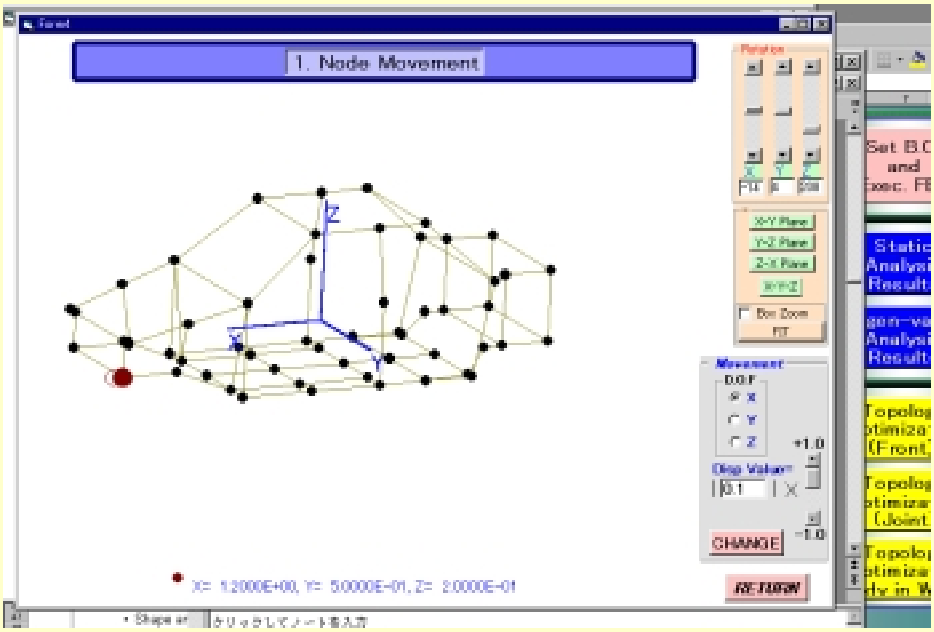

h
w
Selection
Point_Create
_Move
Line_Create
_Delete
Exit
width: 0.2, height: 0.2
Typical shapes
Width
Height
Thick
Type 1
Type 2
Type 3
Width
0.15
Height
0.15
Thickness
0.001
Reset and Create
Width
1
Model Disp
Tort. Stiff


Torsional_Stiffness = 7.9644E+06
Animate
Disp Scale:
Select Case Number:
Max Value = 1.1898E-02
Selection of "Element Forces" and "Strain Energy"
px py pz mx my
Strain_Energy
Positive
Negative
END
Tort. Stiff.


Set B.C. and Exec. FEA
Static Analysis Results
Eigen-value Analysis Results
Topology Optimization (Front)
Topology Optimization (Joint)
Topology Optimization (Body in White)



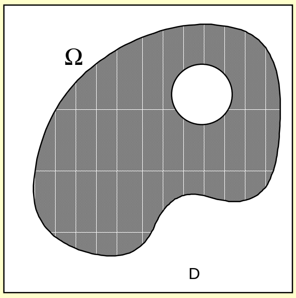
Selection Window
1.節点移動
2.節点追加
3.要素追加
4.要素削除
5.物性/質量変更
6.結合部作成
7.境界条件
Set B.C. and Exec. FEA
Static Analysis Results
Eigen-value Analysis Results
Topology Optimization (Front)
Topology Optimization (Joint)
Topology Optimization (Body in White)



1. Node Movement
X-Y Plane
Y-Z Plane
Z-X Plane
X-Y-Z
Box Zoom
Fit
Movement
D.O.F
Disp Value
CHANGE
RETURN
X= 1.2000E+00, Y= 5.0000E-01, Z= 2.0000E-01

Extension to Continuum

Characteristic Function



Ω = unknown optimum domain
D = specified fixed domain

$$\chi_{\Omega}(x) = \begin{cases} 1 & \text{if} \quad x \in \Omega \\ 0 & \text{if} \quad x \notin \Omega \quad \text{i.e.} \quad x \in D \setminus \Omega \end{cases}$$

What can we get from this ?

Optimal Material Distribution

Strain Energy of a Body

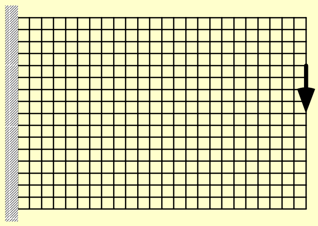
$$U = \frac{1}{2}\int_{\Omega} \boldsymbol{\varepsilon}^T \underbrace{\mathbf{D}\boldsymbol{\varepsilon}}_{=\boldsymbol{\sigma}} \, d\Omega = \frac{1}{2}\int_{D} \boldsymbol{\varepsilon}^T \underbrace{\chi_{\Omega}\mathbf{D}}_{=\mathbf{D}_{new}} \boldsymbol{\varepsilon} \, dD$$

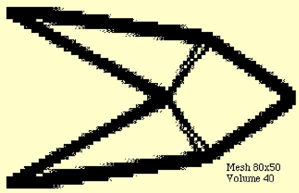
Shape Design — Material Design

Find the best Ω → Find the best $\mathbf{D}_{\text{new}}$

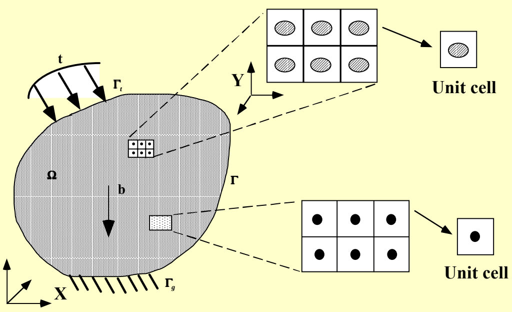
Homogenization Design Method

- Shape and Topology Design of Structures is transferred to Material Distribution Design (Bendsoe and Kikuchi, 1988)





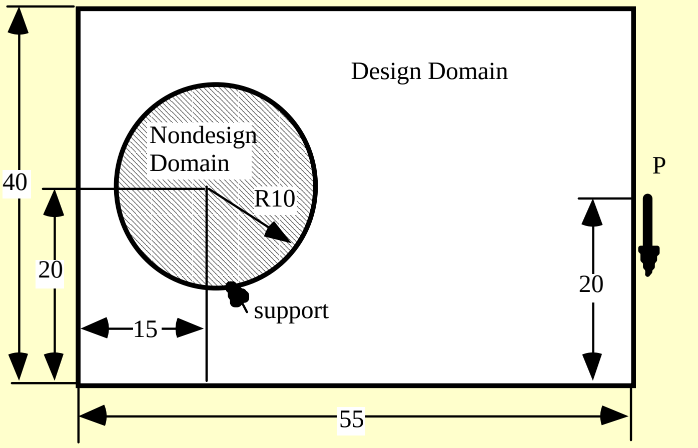
Homogenization Method : Mathematics



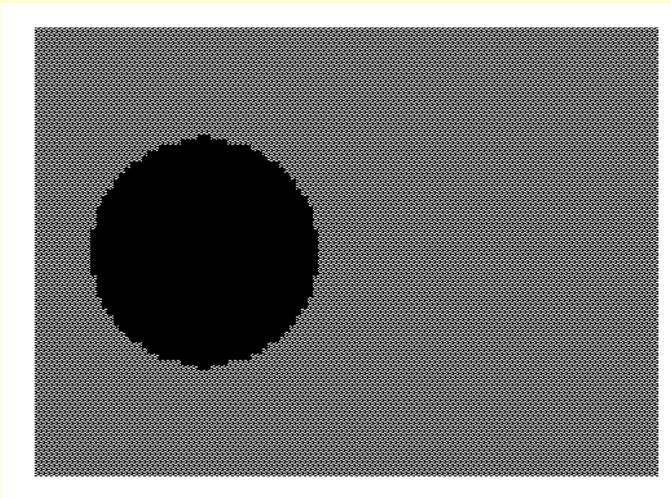
Review

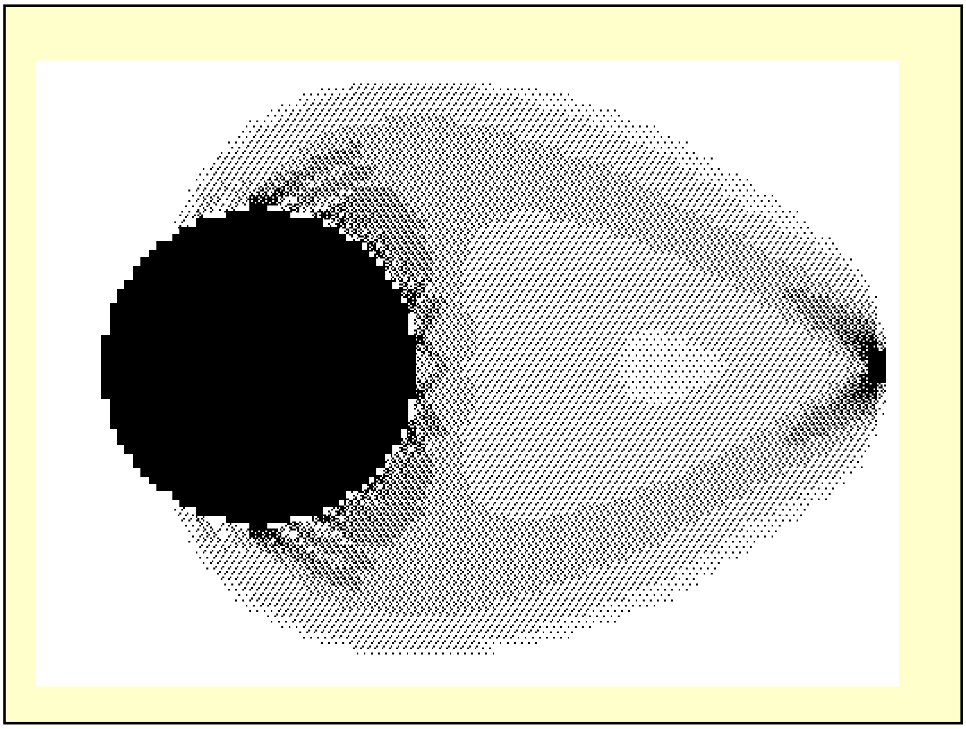
- **Under the assumption of periodic microstructures which can be represented by unit cells.**
- **Using the asymptotic expansion of all variables and the average technique to determine the homogenized material properties and constitutive relations of composite materials.**

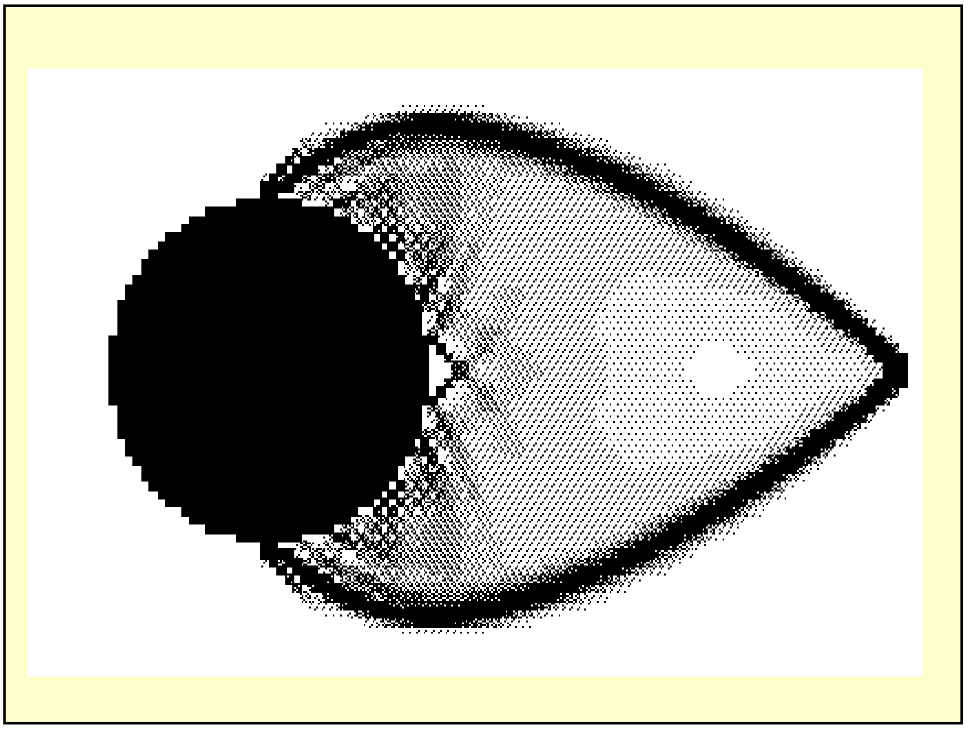
HDM Test Problem

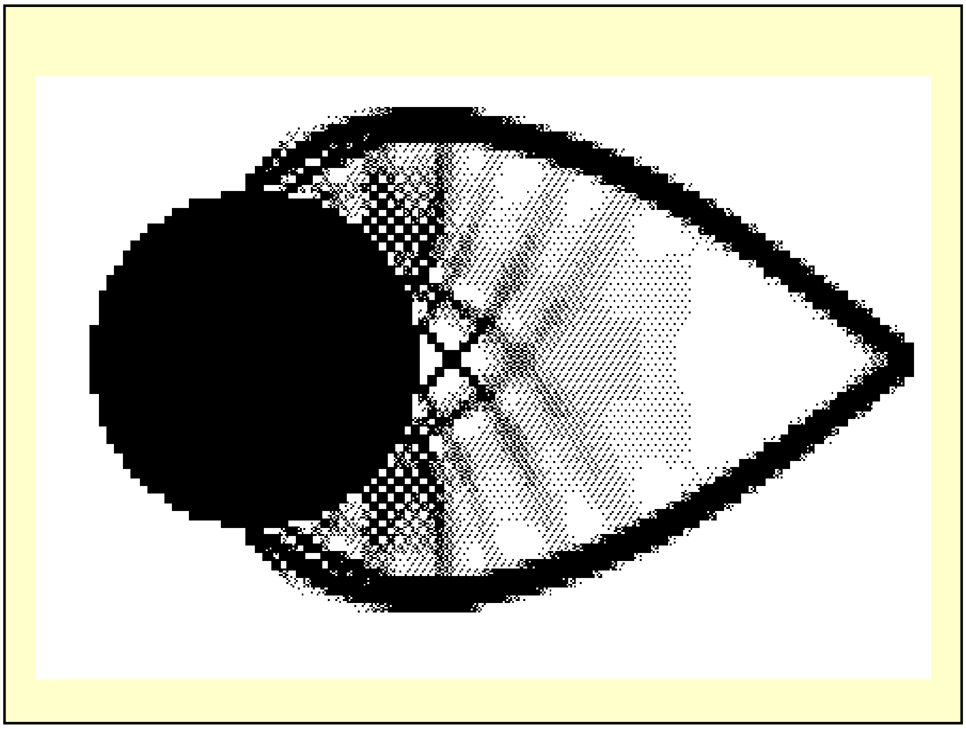


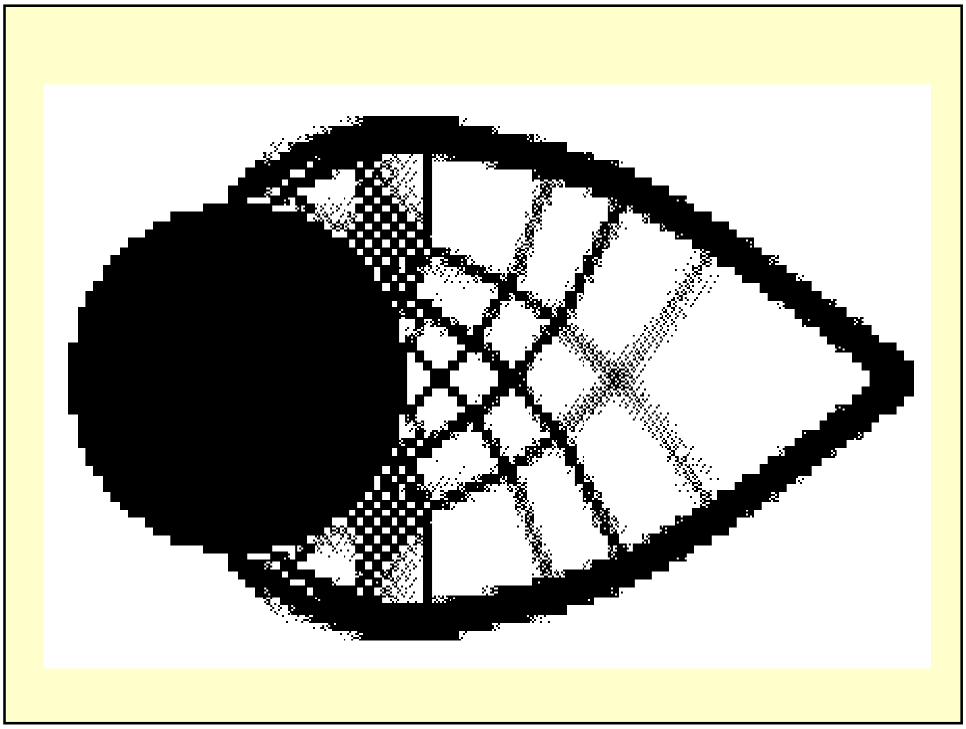
Starting from Uniform Perforation

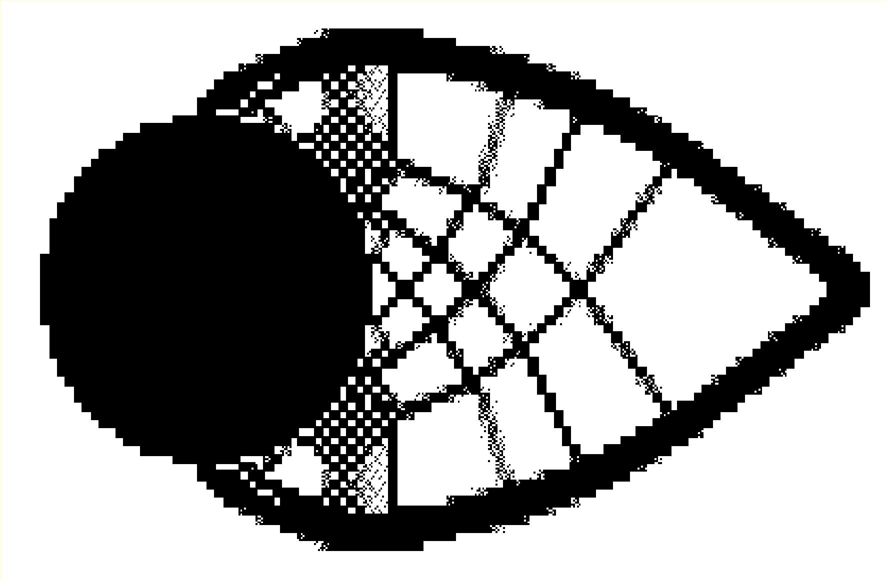






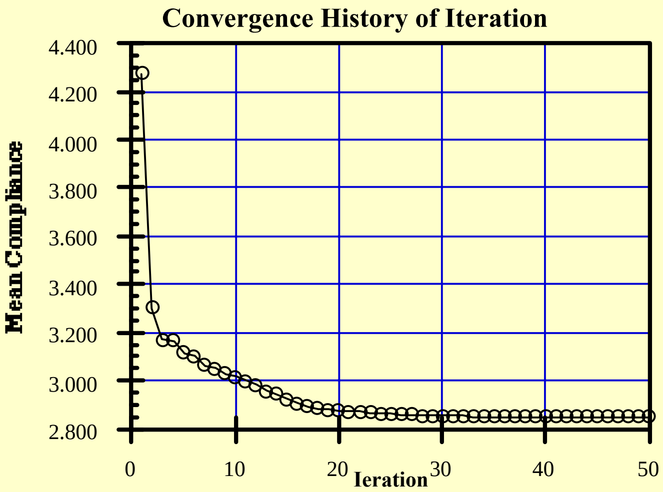




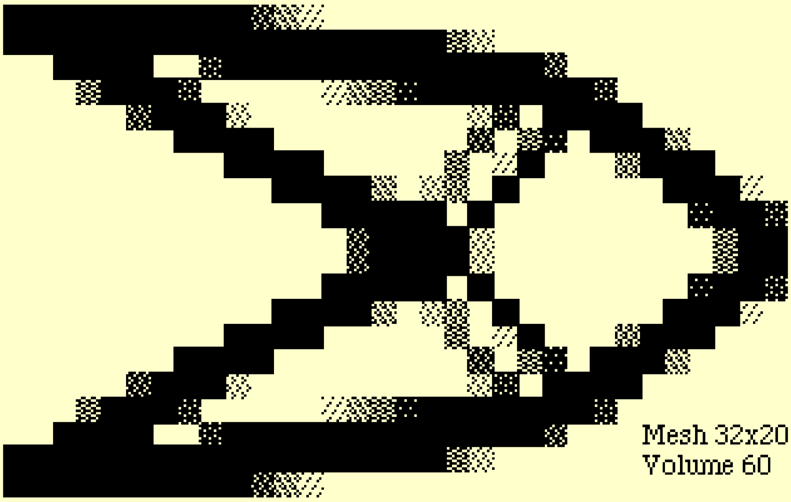


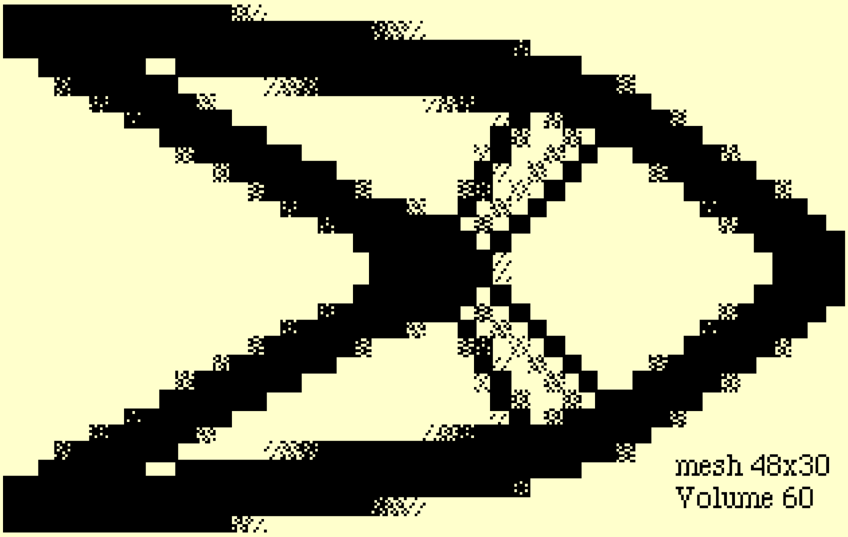
Design Process

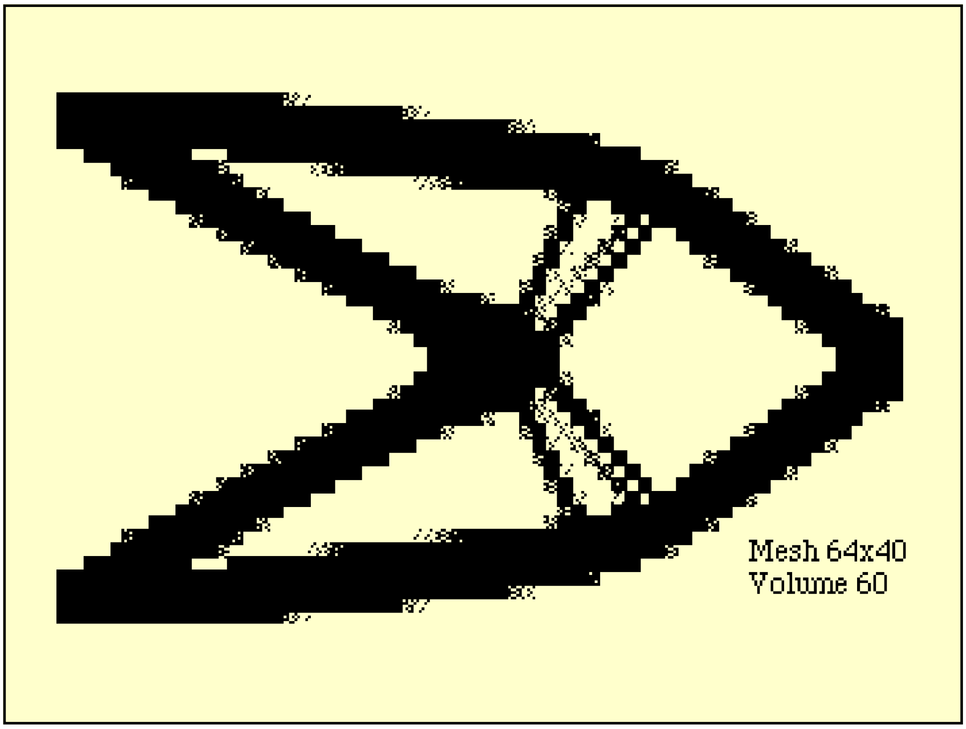
Structural Formation Process

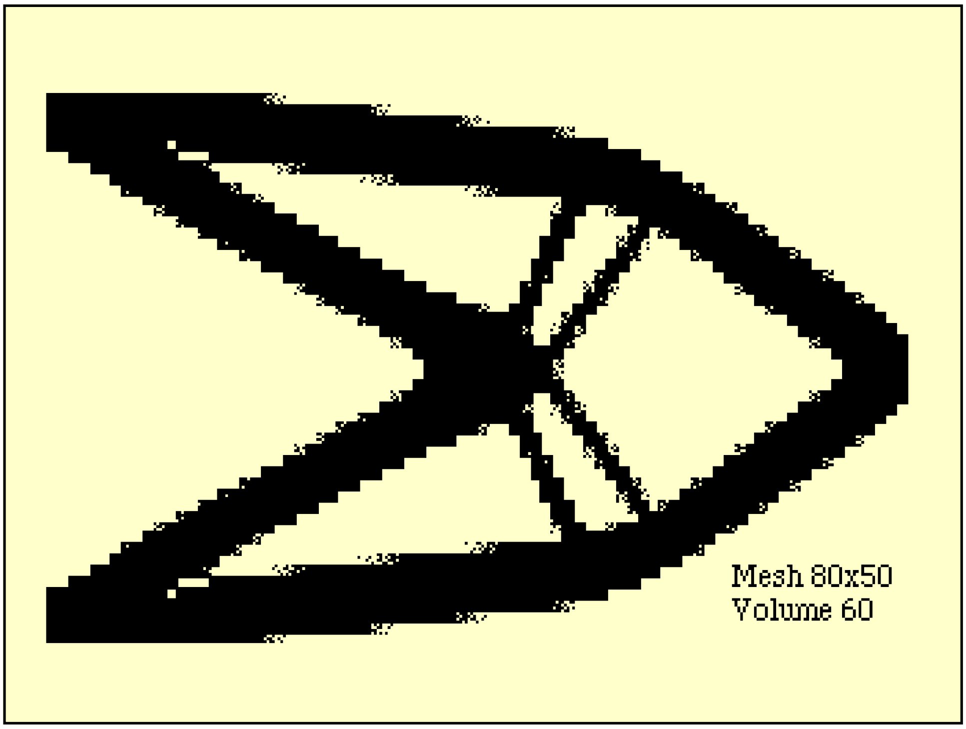


Mesh Refinement

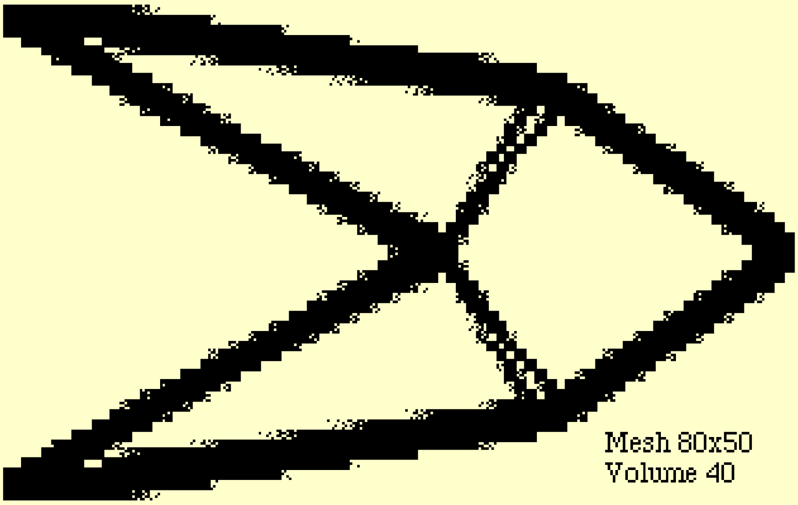


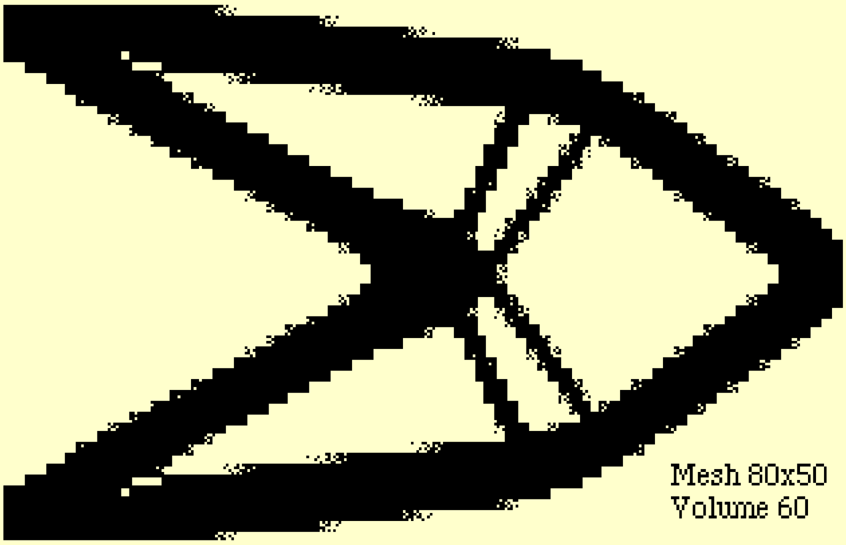


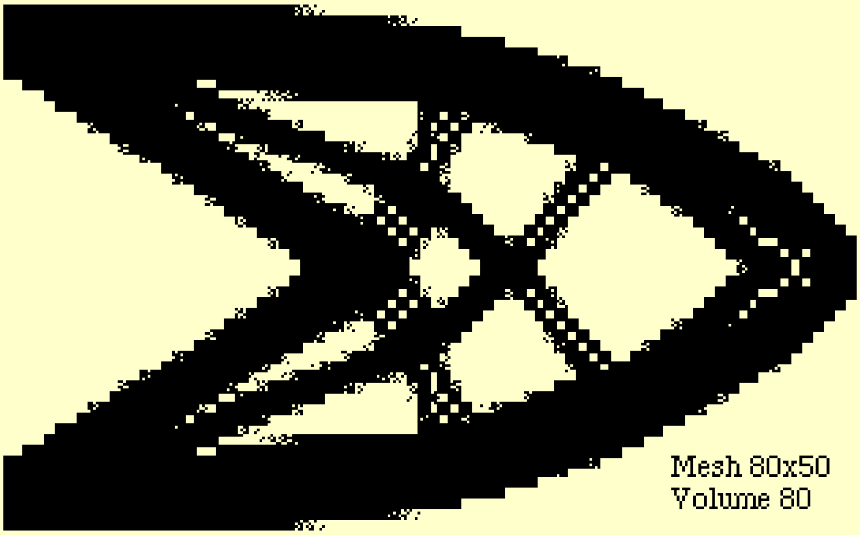

Mesh 64x40
Volume 60


Mesh 80x50
Volume 60

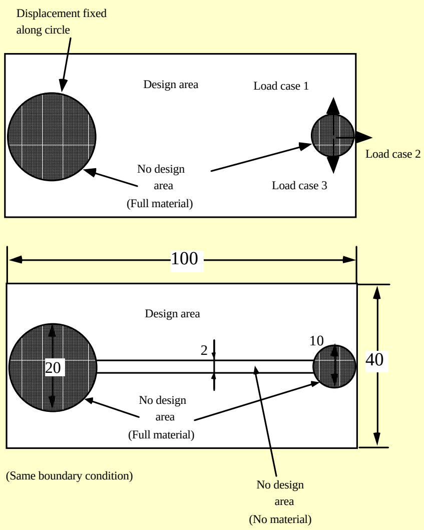
Change Volumes



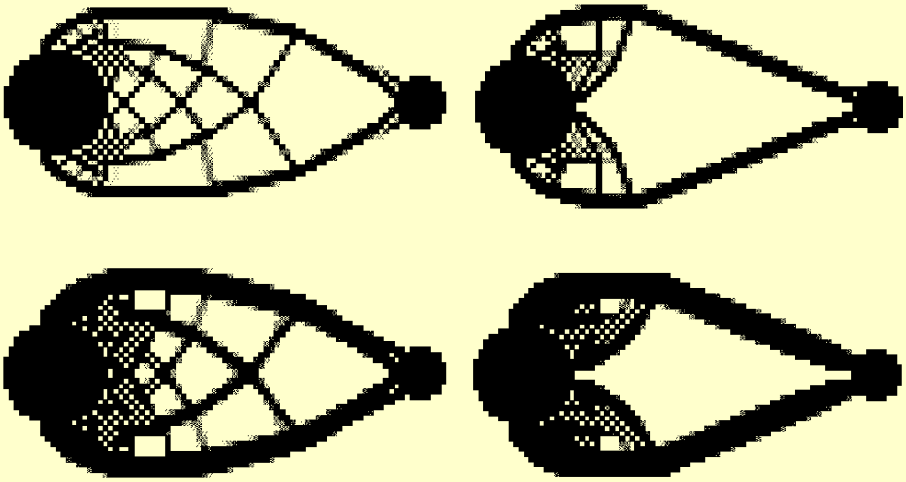




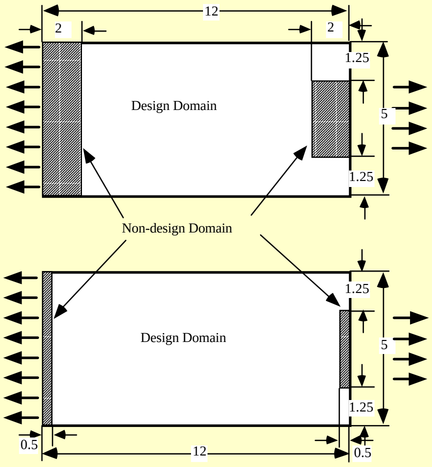
Design Constraint



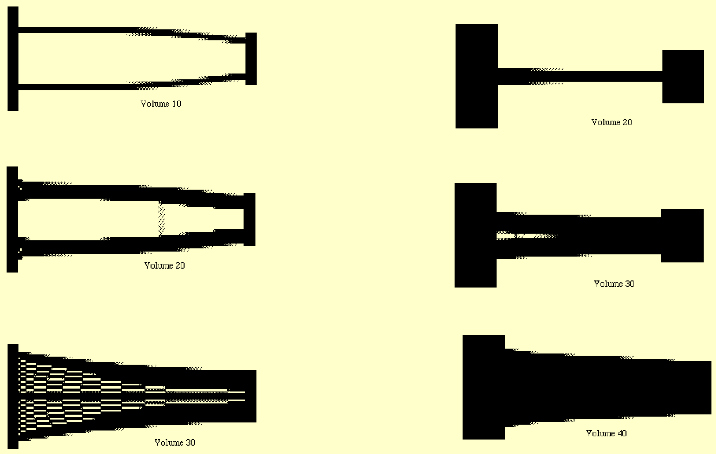
Result of Design Constraint



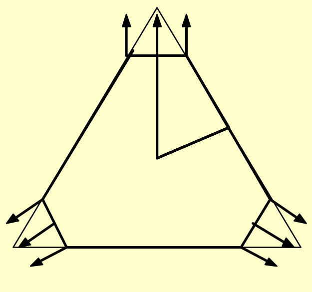
Influence of Design Domain

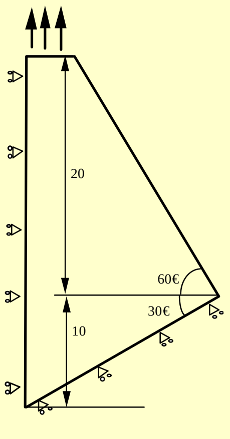


Different Topology

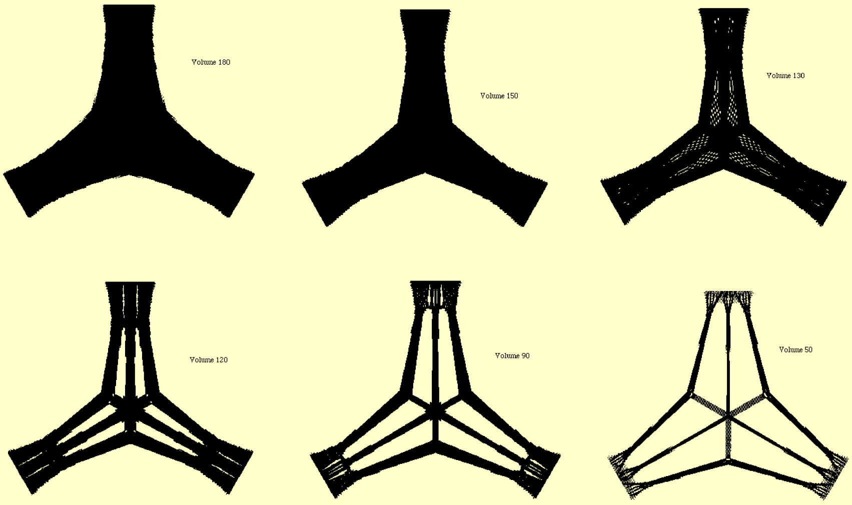


Shape Design Example



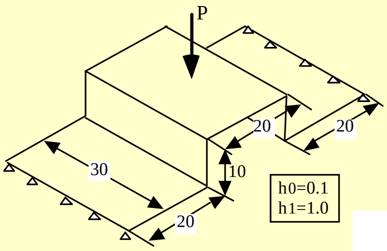


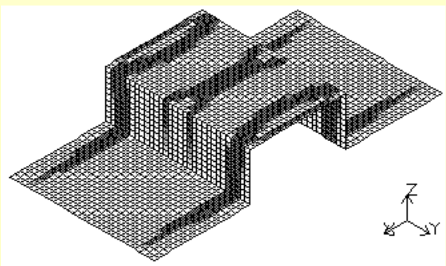
Shape to Topology



Extension to Shells

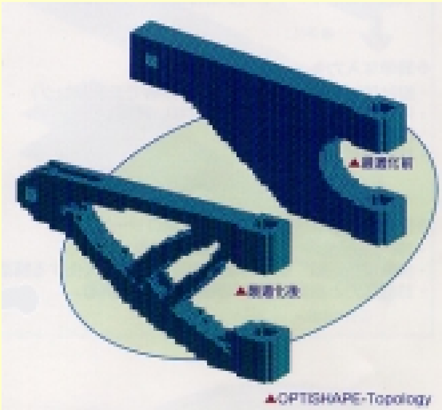
Rib Formation





Commercialization of HMD

From University to Industry



Three-dimensional shaping of a structure for Optimum without any spline functions

OPTISHAPE Development
1986~1989

Acceptance

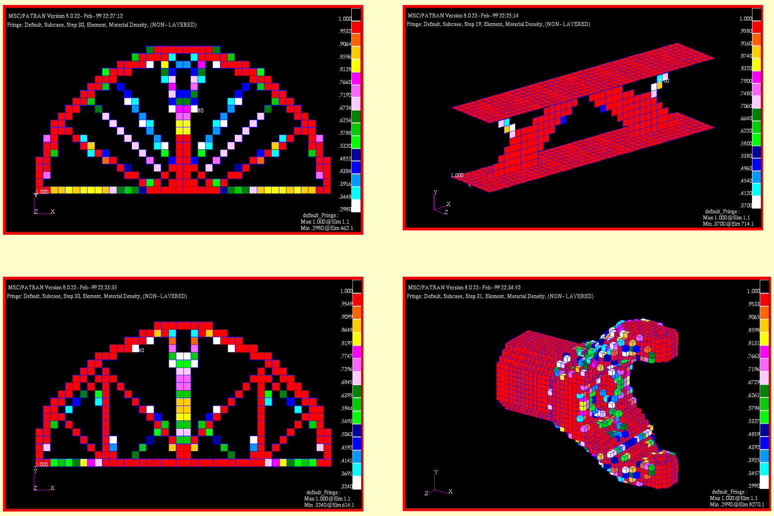
Topology Optimization Methods

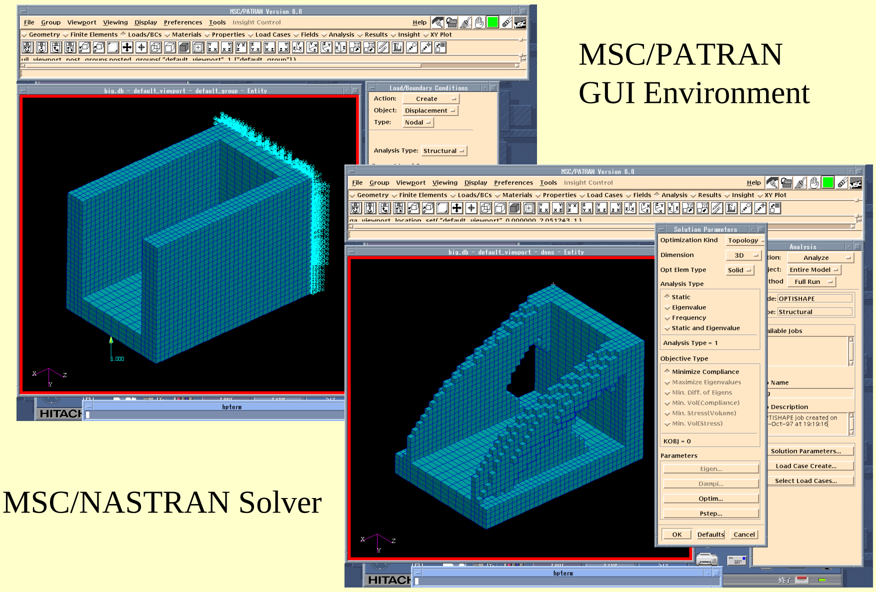
- Commercial Codes have been developed in USA, Europe, and Pacific Regions
- OPTISHAPE@Quint Corporation, Tokyo, Japan, 1989
- OPTISTRUCT@Altair Computing, Troy, USA, 1996
- MSC/CONSTRUCT@MSC German, 1997
- And Others (OPTICON, ANSYS, …..)

MSC/NASTRAN-OPTISHAPE

- Quint/OPTISHAPE + MSC/NASTRAN
- Shape and Topology Optimization
  - Static Global Stiffness Maximization
  - Maximizing the Mean Eigenvalues
    - Frequency Control for Free Vibration
    - Increase of the Critical Load
- MSC/PATRAN integration
- Developed by MSC Japan and Quint Corp.

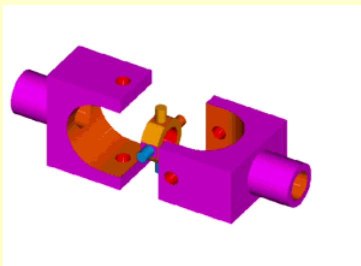
Static/Dynamic Stiffness Maximization

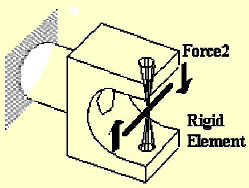


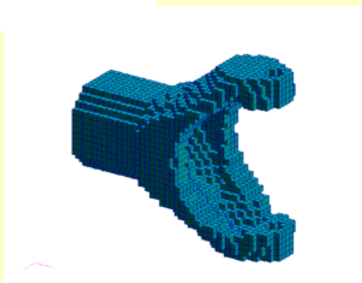


MSC/PATRAN
GUI Environment

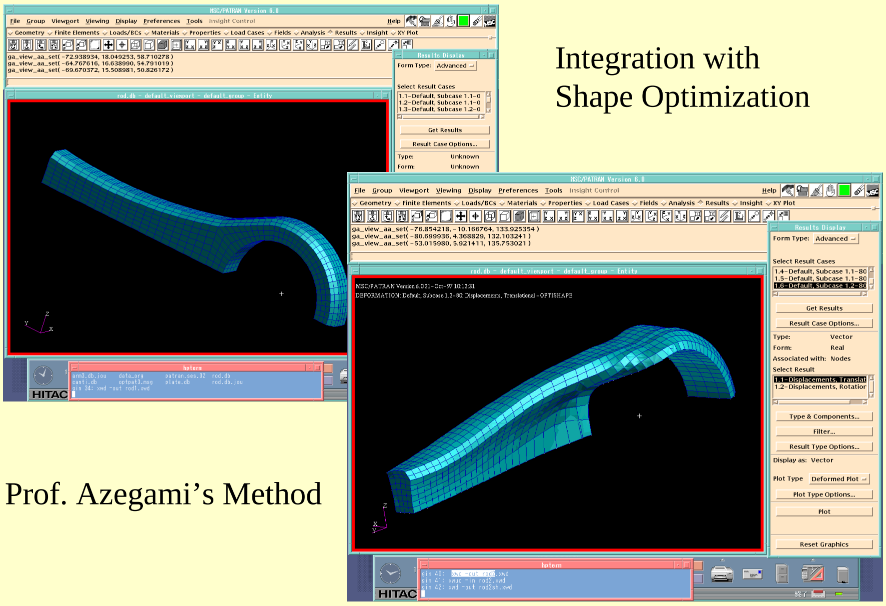
MSC/NASTRAN Solver







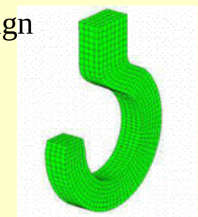
Design Example by MSC.NASTRAN-OPTISHAPE

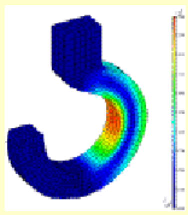


Integration with
Shape Optimization

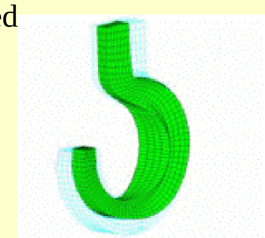
Prof. Azegami's Method

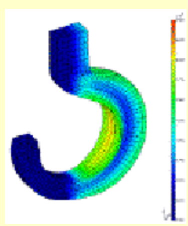
Initial Design



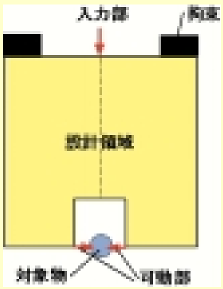


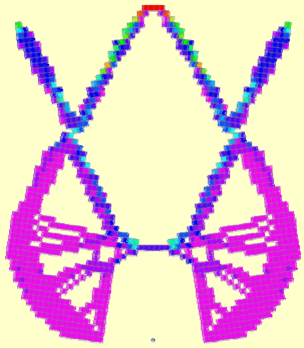
Optimized





Shape Design Optimization by MSC.NASTRAN-OPTISHAPE

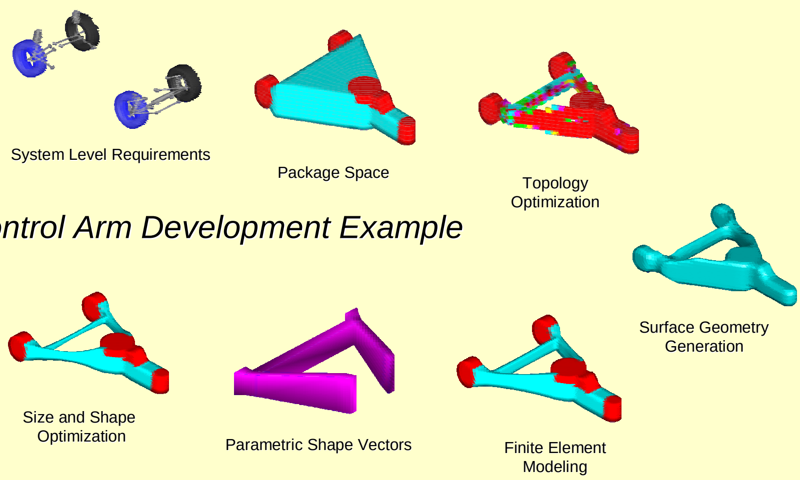




Compliant Mechanism Design by QUINT/OPTISHAPE

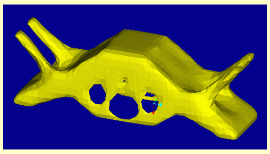
Application of QUINT/OPTISHAPE @ Kanto Automotive

Altair: Concept Design Environment
Product Design Synthesis

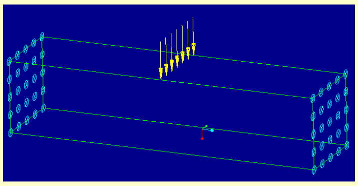


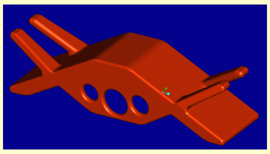
Altair/OptiStruct

- Input:
  - FE model of design space
  - Load cases, frequencies, constraints
  - Mass target



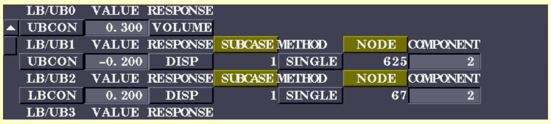
- Output:
  - Optimal material distribution via ‘density’ plot
  - CAD geometry interpretation : using OSSmooth
- Then…use to create optimal design





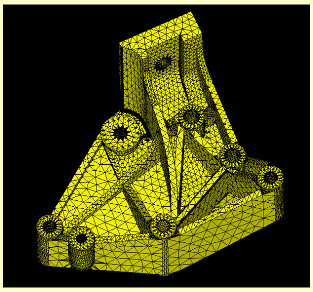
OptiStruct Version 3.4

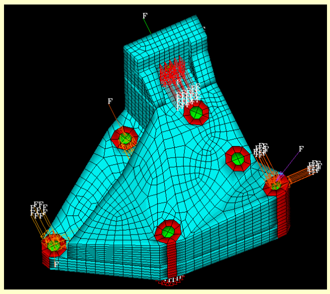
- Expanded Objective function
  - Minimize Mass, Stiffness or Frequency
  - Constraints on Mass, Stiffness, Freq, Disp
- Now available on Windows NT
- FE improvements, faster solution time
- HTML/Windows on-line documentation
- Improved integration with HyperMesh3.0



OptiStruct Case Study
Volkswagen Bracket

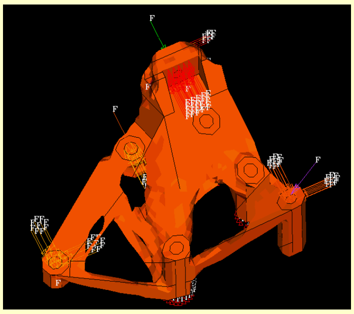
- Minimize Mass of Engine Bracket
  - Subject to stiffness/frequency constraints
- 7 loadcases: operating, pulley, transport

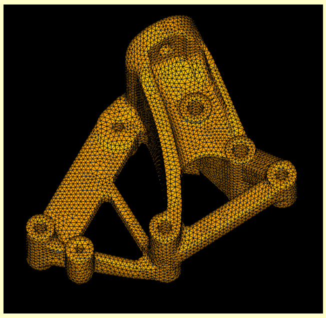




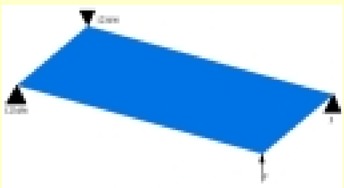
OptiStruct Case Study
Volkswagen Bracket Results

- Mass reduced by 23%
  - Original mass 950g ; Final mass 730g
- Performance targets were met

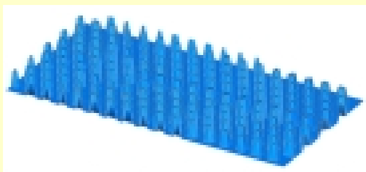




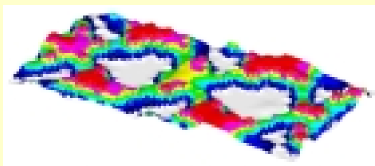
OptiStruct: Topography Design
for Future Automotive Body Engineering



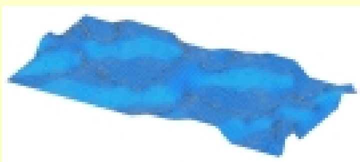
Loads and Constraints



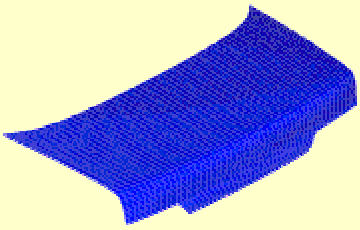
Shape variables for Topography Optimization (symmetric)

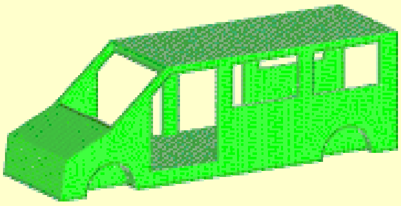


OptiStruct solution (colors show rib height)



Finite element model of OptiStruct solution



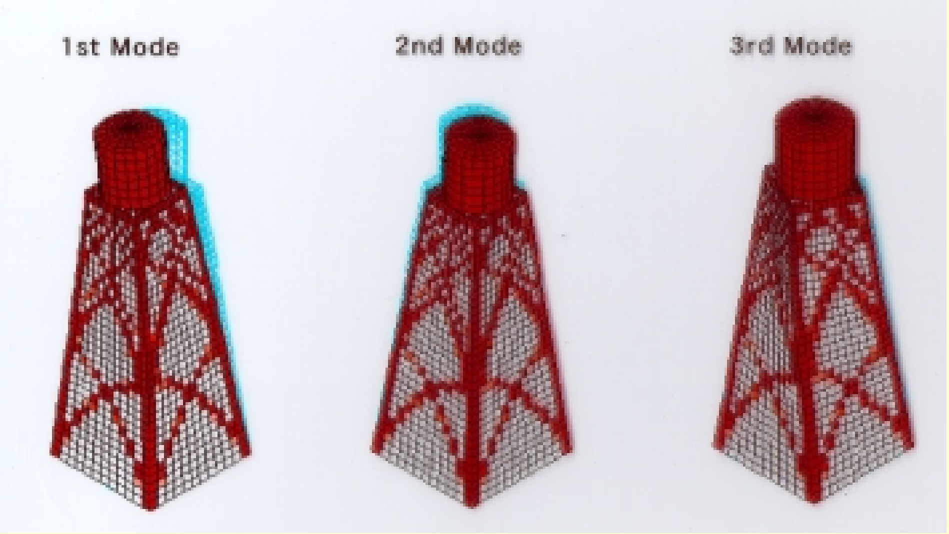


ALTAIR/OPTISTRUCT Results

Extension of HDM

Topology Optimization Method

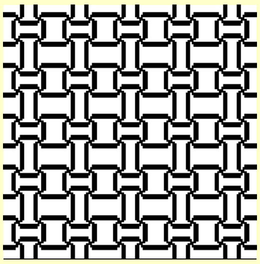
- Structural Design
  - Static and Dynamic Stiffness Design
  - Control Eigen-Frequencies
  - Design Impact Loading
  - Elastic-Plastic Design
- Material Microstructure Design
  - Young’s and Shear Moduli, Poisson’s Ratios
  - Thermal Expansion Coefficients
- Flexible Body Design (MEMS application)
- Piezocomposite and Piezoelectric Actuator Design

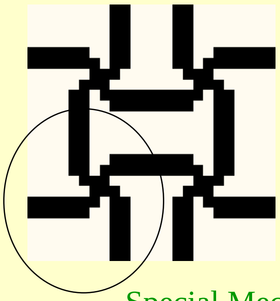


QUINT/OPTISHAPE Application to Contro Frequencies

Material Design

Special Mechanism : Negative ν

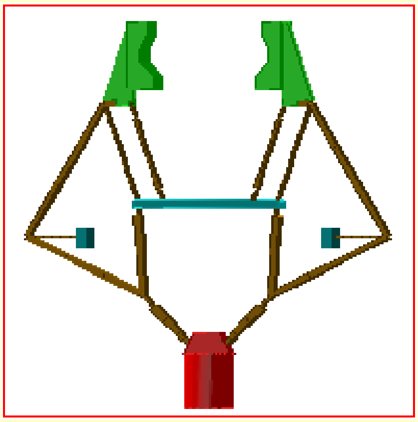




Special Mechanism

Compliant Mechanism Design

Professor S. Kota @ UM



Negative Thermal Expansion

Bing-Chung Chen's Design

$$\boldsymbol{\beta}^H = \begin{bmatrix} -8.01 & 0 \\ 0 & -7.89 \end{bmatrix}$$

$$\boldsymbol{\alpha}^H = \begin{bmatrix} -52.7 & 0 \\ 0 & -58.9 \end{bmatrix}$$