Response Surface Methodology and Its application to automotive suspension designs

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Outline

I. Introduction & Basis of RSM
   1. History of RSM
   2. What's RSM
   3. Why is RSM
   4. Least square method
   5. Design Of Experiment (DOE)

II. Its application to automotive suspension designs
   1. Size optimization for beam stiffens
   2. Size optimization for section beam property

I. Introduction and Basis of Response surface Methodology (RSM)

History of RSM

- 1951 Box & Wilson

  Contributed RSM of Quadratic Polynomials
- 1988 ~ 1990 Design Of Experiments (DOE)

  Taguchi DOE (Taguchi Method)

  Myers & Montgomery DOE → RSM
- 1992 ~ 1994 Quality engineering

  The Process of a semiconductor … etc
- 1995 Optimization for Numerical Analysis

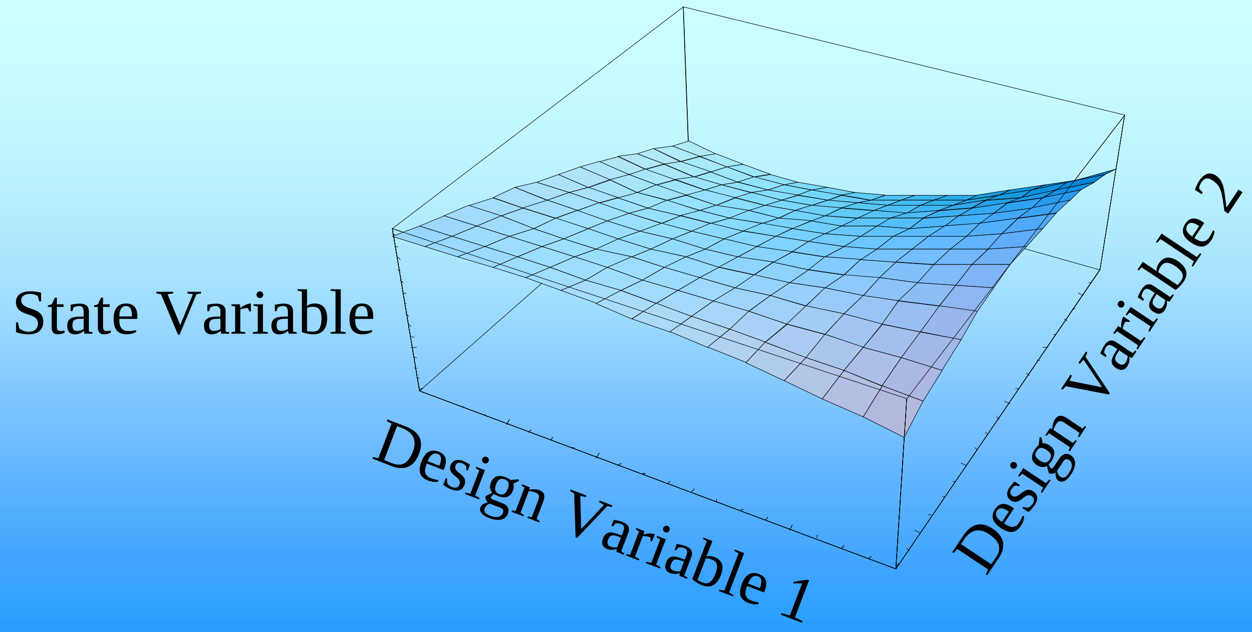
  Haftka Composite Wing Structural Optimization

  Shiratori Design Optimization of automotive Seat frame

What's RSM

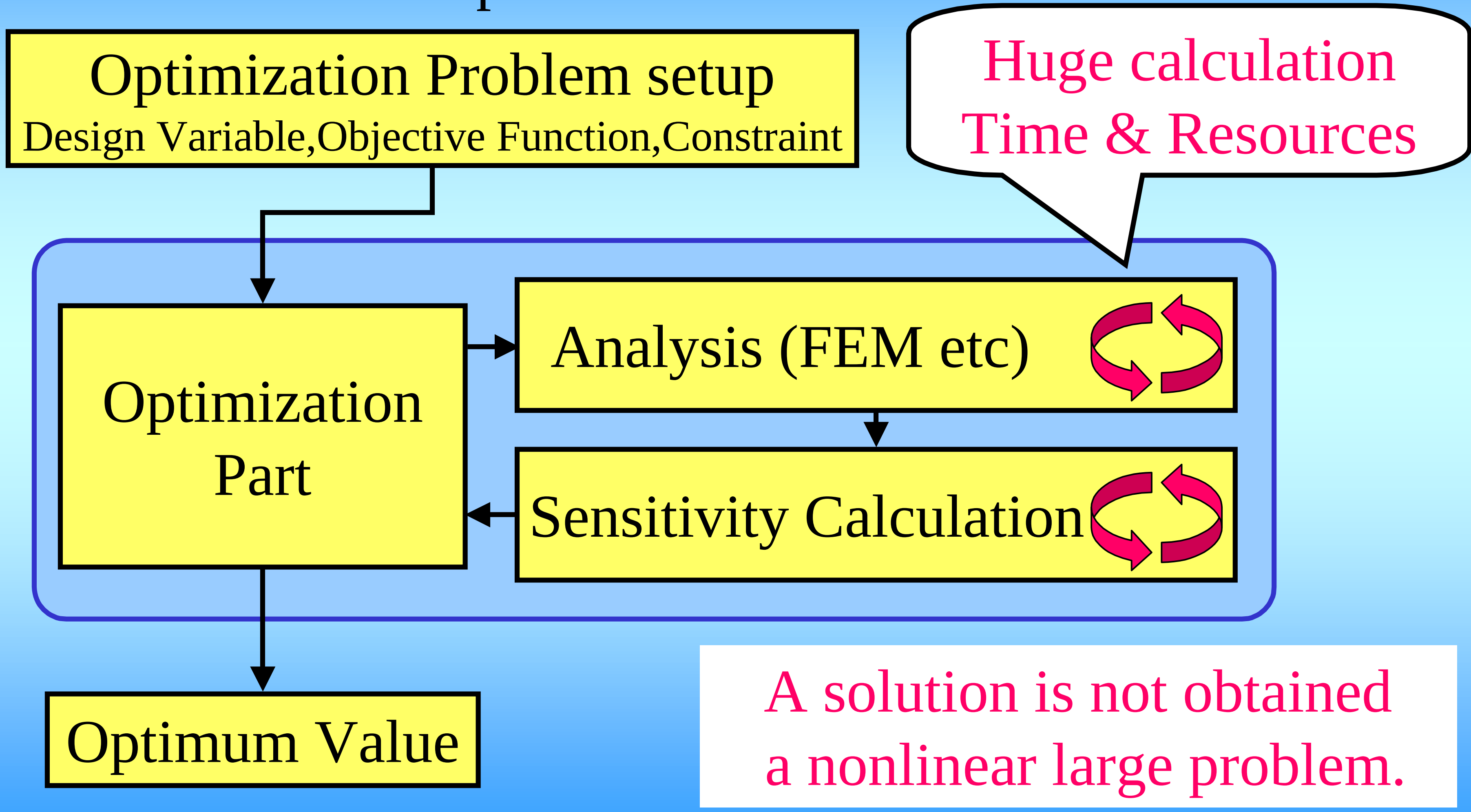
- Approximation Optimization
- Approximation Function = Response surface
- Response surface

  Least square method & Design of Experiments



Why is R.S.M.

Conventional Optimization



Why is R.S.M.

Optimization problem setup
Design Variable,Objective Function, Constraint

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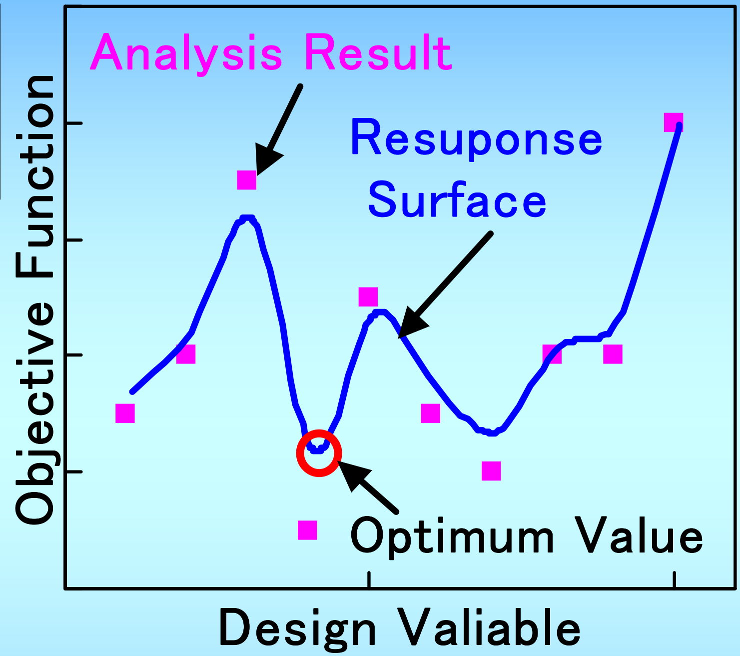
Response surface creation
A function is approximated.

↓

Optimization calculate
Using the Response Surface

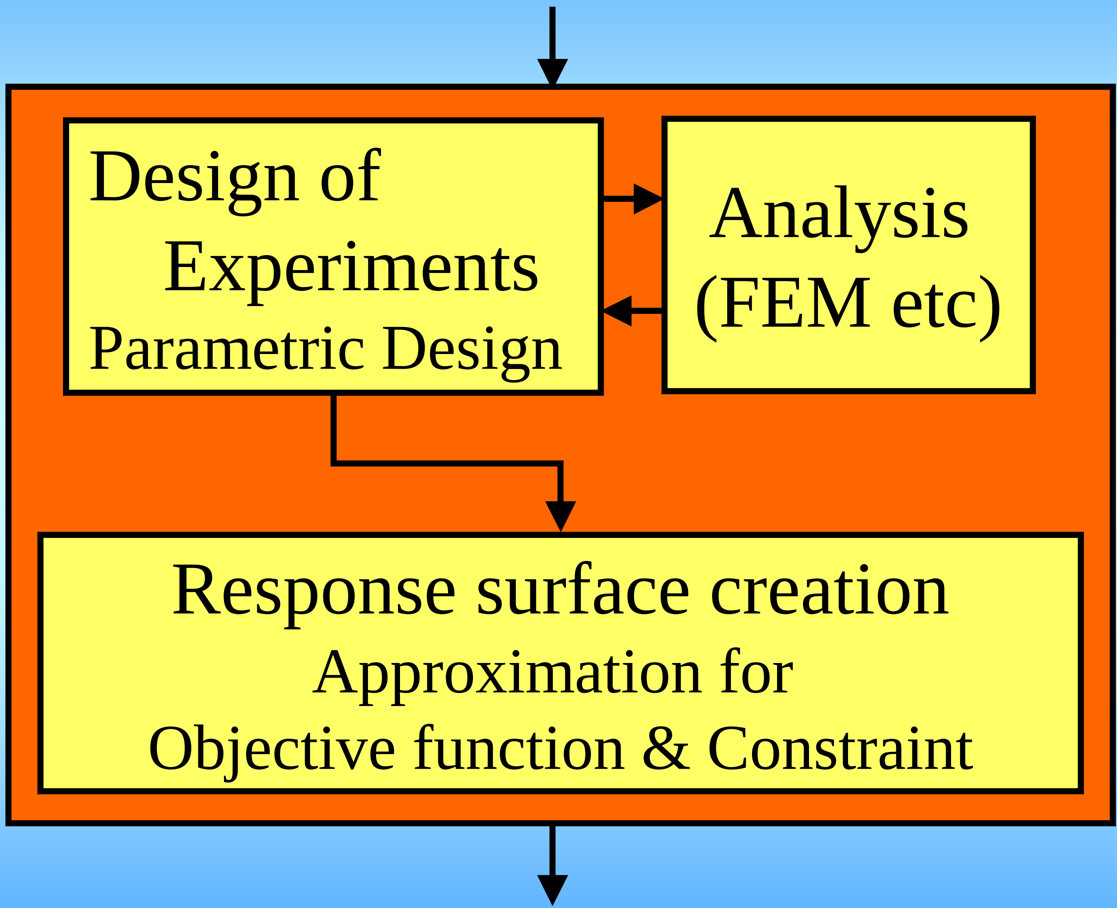
↓

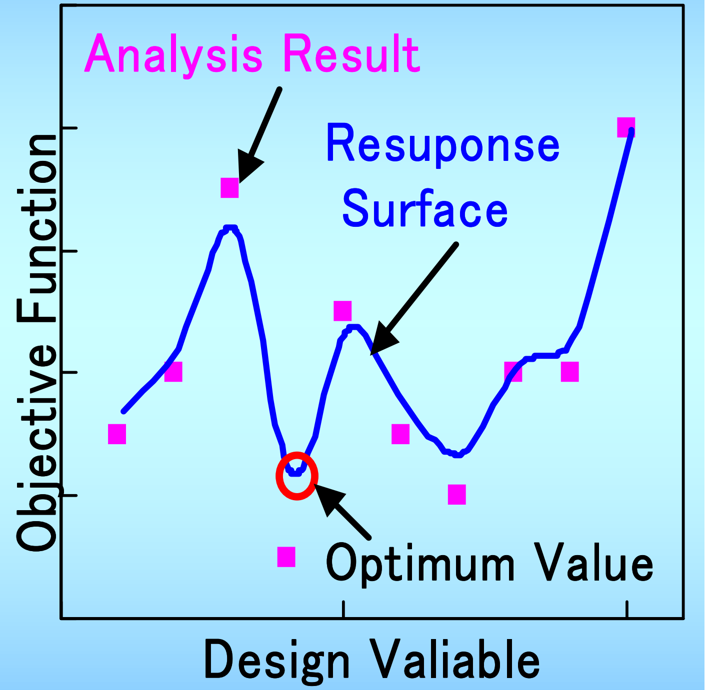
Optimum value



Calculation is very early

Response surface creation





Response surface

Design Variables : $x_i \quad (i = 1 \cdots n)$

State Variable : $y = f(x_1 \cdots x_n) + \varepsilon$

- Least Square Method
  - Polynomials
  - Exponential
  - Logarithm … etc

  } Linearized
- Neural Network
- Spline interpolation
- Lagrange interpolation

Least square method (1)

Used Linear function

Coefficient of function

Evaluation of function

Easily obtained by Statistics

Quadratic polynomials

$$y = \beta_0 + \sum_{i=0}^{n} \beta_i x_i + \sum_{i=0}^{n} \beta_{ii} x_i^2 + \sum_{i<j}^{n} \beta_{ij} x_i x_j$$

Example of two variables for simplification

$$y = \beta_0 + \beta_1 x_1 + \beta_2 x_2 + \beta_3 x_1^2 + \beta_4 x_2^2 + \beta_5 x_1 x_2$$

$$x_1^2 = x_3 \,,\; x_2^2 = x_4 \,,\; x_1 x_2 = x_5 \quad \rightarrow \quad \textit{Linearized}$$

Least square method (2)

$$\boldsymbol{y} = \boldsymbol{X}\boldsymbol{\beta} + \boldsymbol{\varepsilon}$$

Number of Experiments : n
Number of variable : k

$$\boldsymbol{y} = \begin{bmatrix} y_1 \\ y_2 \\ \vdots \\ y_n \end{bmatrix} \quad \boldsymbol{X} = \begin{bmatrix} 1 & x_{11} & x_{12} & \cdots & x_{1k} \\ 1 & x_{21} & x_{22} & \cdots & x_{2k} \\ \vdots & \vdots & \vdots & \ddots & \vdots \\ 1 & x_{n1} & x_{n2} & \cdots & x_{nk} \end{bmatrix} \quad \boldsymbol{\beta} = \begin{bmatrix} \beta_1 \\ \beta_2 \\ \vdots \\ \beta_n \end{bmatrix} \quad \boldsymbol{\varepsilon} = \begin{bmatrix} \varepsilon_1 \\ \varepsilon_2 \\ \vdots \\ \varepsilon_n \end{bmatrix}$$

Error Sum of Square : $L = \boldsymbol{\varepsilon}^T \boldsymbol{\varepsilon}$ → Minimize

Least square estimations of β : $\boldsymbol{b}$

$$\boldsymbol{b} = \left(\boldsymbol{X}^T \boldsymbol{X} \right)^{-1} \boldsymbol{X}^T \boldsymbol{y}$$

Least square method (3)

Total Sum : $T = \sum_{i=1}^{n} y_i$

Sum Square of Error : $SSE = \boldsymbol{y}^{\boldsymbol{T}} \boldsymbol{y} - \boldsymbol{b}^{\boldsymbol{T}} \boldsymbol{X}^{\boldsymbol{T}} \boldsymbol{y}$

Sum Square of Regression : $SSR = \boldsymbol{b}^{\boldsymbol{T}} \boldsymbol{X}^{\boldsymbol{T}} \boldsymbol{y} - T^2 / n$

Total Sum of Square : $Syy = \boldsymbol{y}^{\boldsymbol{T}} \boldsymbol{y} - T^2 / n$

Coefficient of multiple determination : R^2

$$R^2 = SSR / Syy = 1 - SSE / Syy$$

Adjusted Coefficient of multiple determination : $R_{ad}{}^2$

$$R_{ad}^2 = 1 - \frac{SSE / (n-k-1)}{Syy / (n-1)}$$

Design Of Experiments (DOE)

Parameter design for efficient experiment

= for obtained better regression formulation

↓

Response surface by least square method

➡ Minimize coefficient variance

Variance covariance matrix ($V(\boldsymbol{b})=cov(b_i,b_j)$)

by least square estimations $\boldsymbol{b}$

$$V(\boldsymbol{b}) = \sigma^2 \left(\boldsymbol{X}^T \boldsymbol{X}\right)^{-1}$$

σ^2 : Error Variance of state variable $\boldsymbol{y}$ → Unknown

➡ Minimize for diagonal of $(\boldsymbol{X}^T \boldsymbol{X})^{-1}$

Orthogonal design

- Mainly used for linear polynomial
- Orthogonal arrays

  Linear → 2-Level factorial design [L8(2^7), L16(2^{15}) …]

  Quadratic → 3-Level factorial design [L9(3^4), L27(3^{13}) …]
- Orthogonal polynomials

  Chebyshev orthogonal polynomials

- Efficient for low order & no interactions

  Large design number for highest-order

  But, very easy

Central Composite Design (CCD)

- Mainly used for quadratic polynomial
- Parametric design

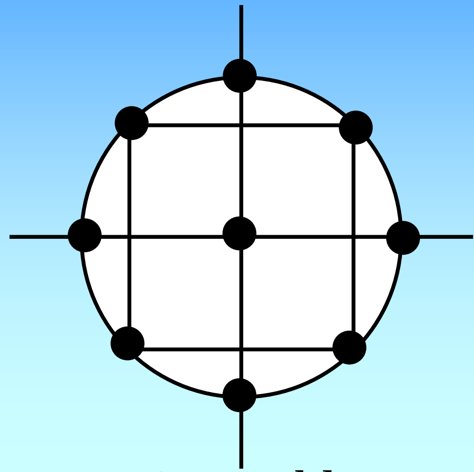
  2-level full factorial design $n_F = 2^k$

  Center point $n_0 > 1$

  Two axial point on axis of each design Variable at distance of design origin. $n_R = 2k$

  Total Number of design $n = 2^k + 2k + n_0$

- No direction dependability = Rotatable design
- Better design for quadratic polynomial



2 variable model

Computer support design

- A-optimality

  Moment matrix : $M = X^T X / n$

  Sum of diagonal value for *M* inverse : $trace(M^{-1})$

  $Minimize[\, trace(M^{-1}) \,]$ Found *X*

  Consideration for diagonal value

  Therefore, Not rotatable design

- D-optimality

  $Maximize[\, M \,]$ Found *X*

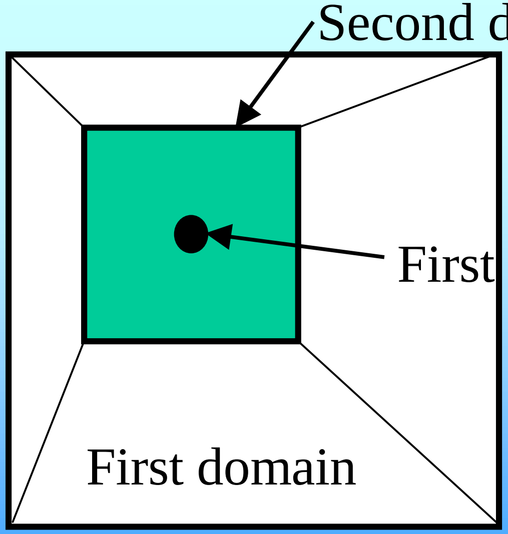
  Consideration for Full value by *M*

  therefore, rotatable design

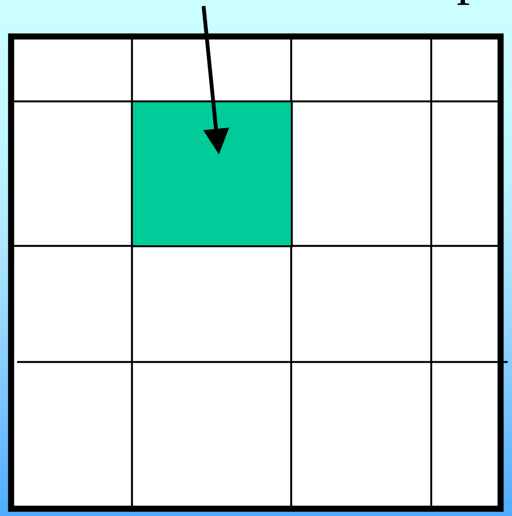
  This method best parametric design. But, difficult

Improvement of RS

- Used high order Polynomial
- Zooming method
- Domain decomposition method
- Kriging model ….. etc



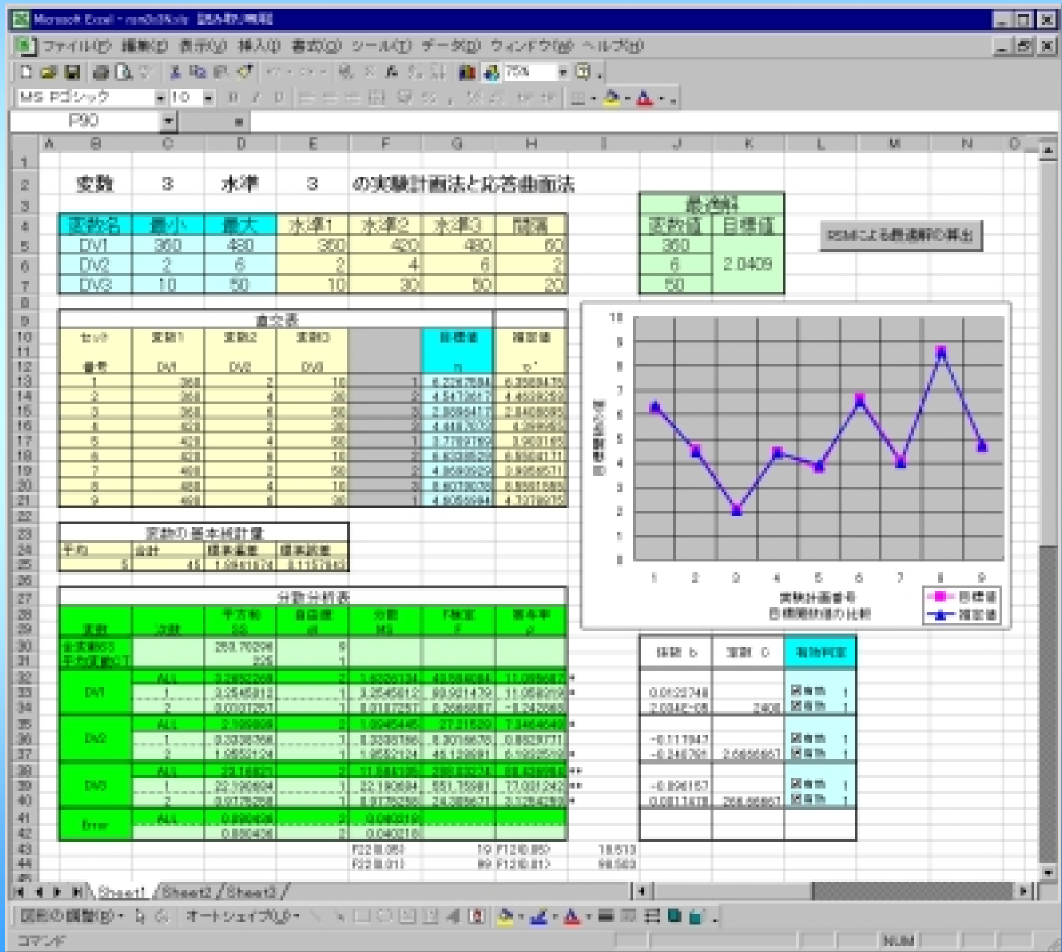
Zooming Method



Domain decomposition Method

RSM Program on Excel

Program for Orthogonal design approach

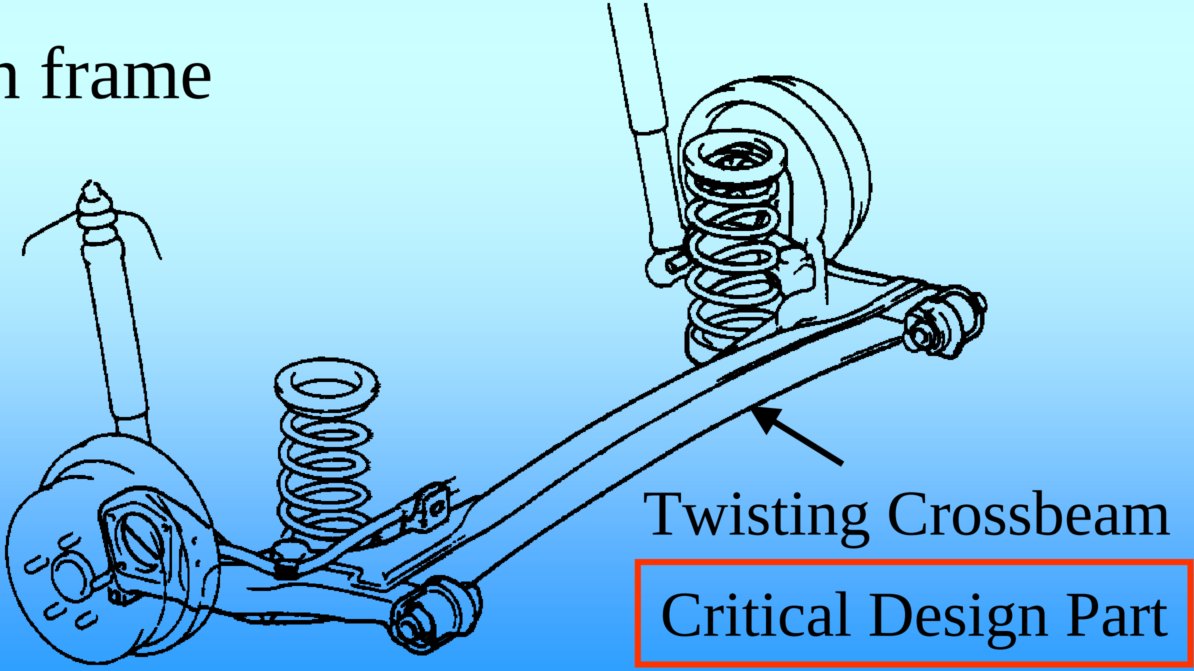


3 factors and
3 levels Design

II. Its application to automotive suspension designs

Trailing Twist Axle Suspension

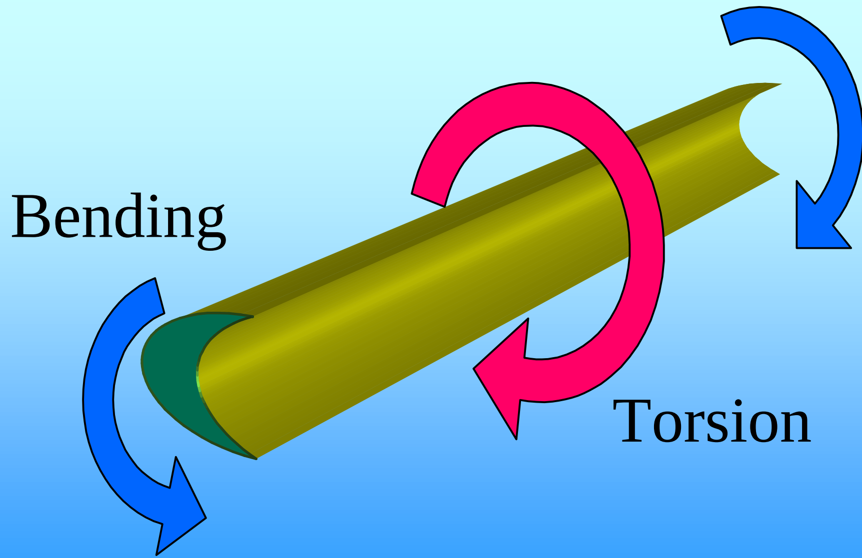
- Mixture of structures and mechanisms
- Good for FF automobiles
- Advantages
  - Simple structures & low cost
  - No suspension frame
  - High stiffness

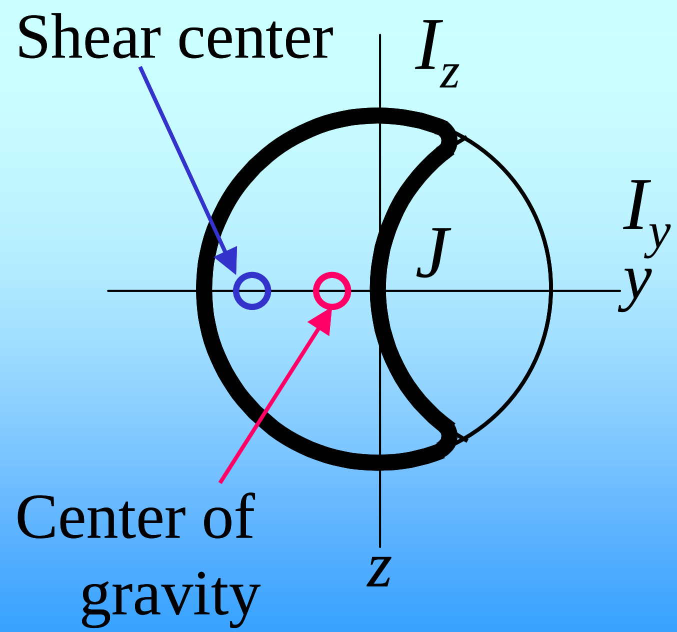


Examples

1) Size optimization for beam stiffens

2) Size optimization for section beam property





1) Size Optimization for stiffness

- Design variable

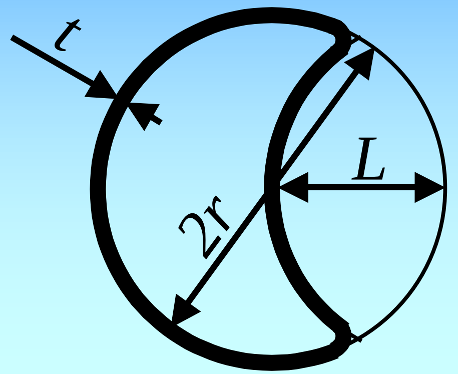
  Thickness : *t* and Forming length : *L*

- Objective function

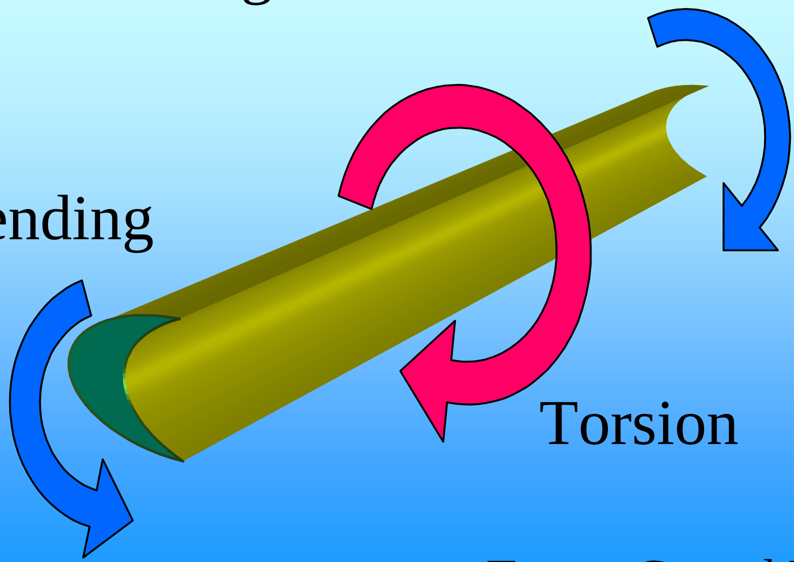
  Minimize total mass

- Constraint

  Torsional and Bending stiffness



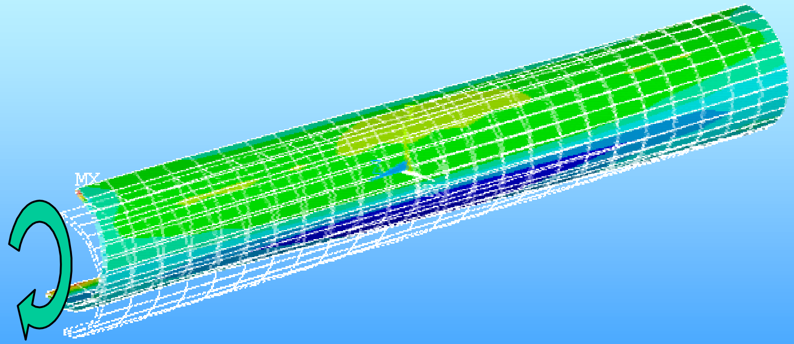
Section of beam



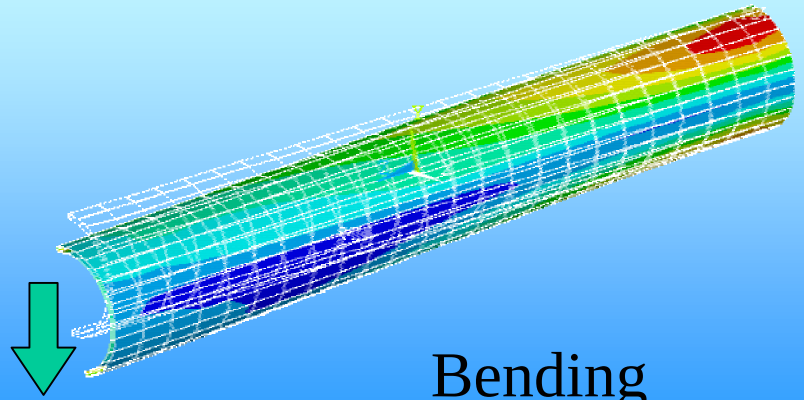
Optimization technique

- Parametric Studies Based on DOE
  - Analysis for FEM
- Optimization used RSM
  - Optimum Design variables

    → Initial design variable for next step
- Optimization used FEM



Torsion



Bending

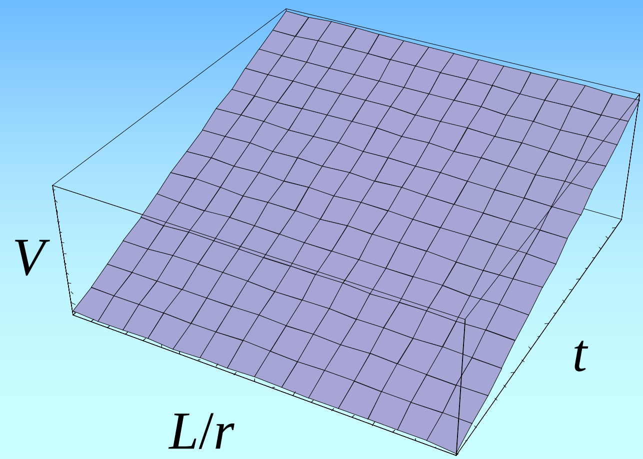
Parametric Studies Based on DOE

- Two factors, three levels in DOE
  - L/r = 0.0,0.5,1.0 where r=50(mm)
  - t=1.0,2.5,4.0 (mm)
- Evaluation function
  - Total volume : V
  - Torsional stiffness : G_T
  - Bending stiffness : G_S

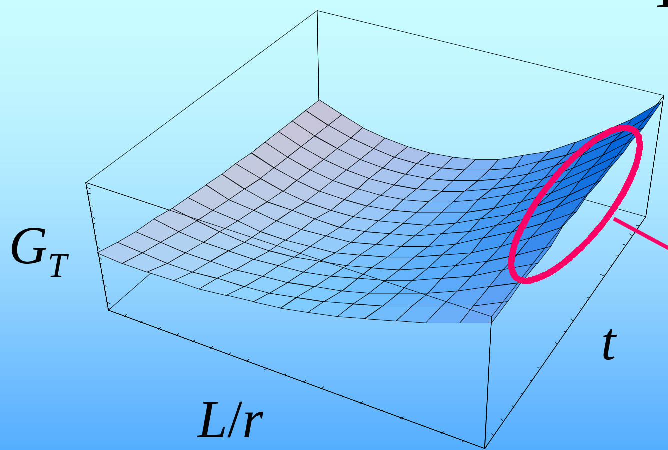
L9(3^4) Orthogonal arrays

	L/r	t
1	0	1
2	0	2.5
3	0	4
4	0.5	1
5	0.5	2.5
6	0.5	4
7	1	1
8	1	2.5
9	1	4

Response surface

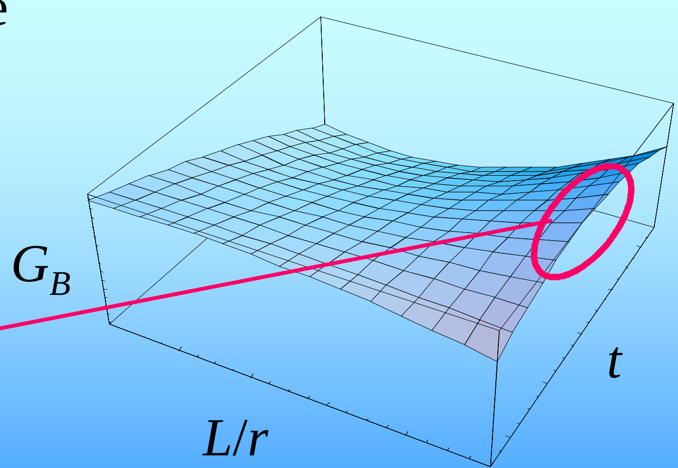


Total volume



Torsion stiffness

Feasible region



Bending stiffness

Optimization Using FEA(Ansys)

Case 1

Initial value (Using RSM) L/r=0.9, t=2.5(mm)

↓ 13 iteration

Optimal Solution L/r=0.987, t= 1.94(mm)

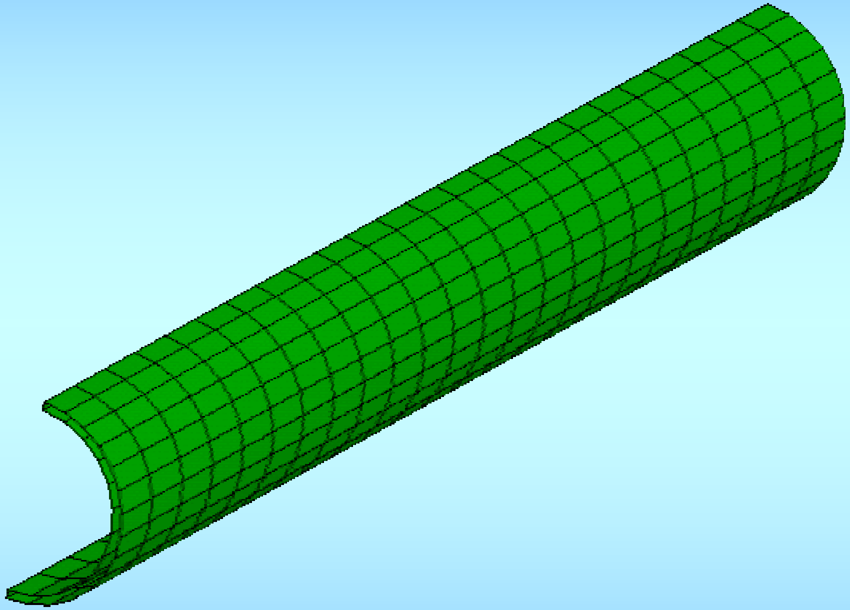
Case 2

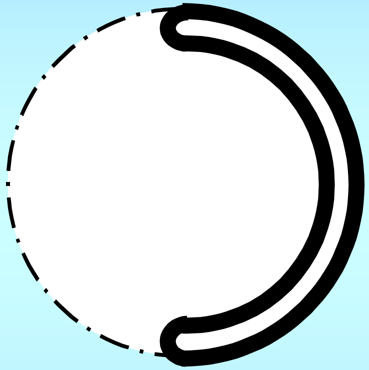
Initial value (Central value) L/r=0.5, t=2.5(mm)

↓ More than 80 iteration

Not Converged

Optimal design





Optimal design $L/r = 0.987$, $t = 1.94$(mm)

2) Optimize for Cross Section Properties

- Design variable

  Thickness : *t* and Forming length : *L*

- Objective function

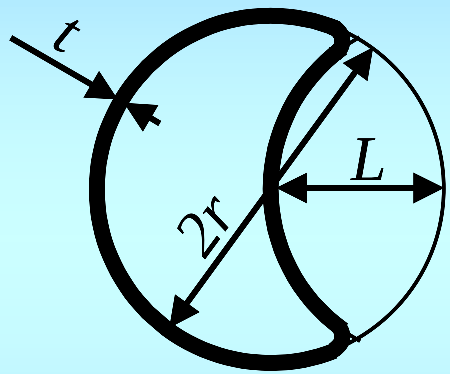
  Minimize Polar moment of Inertia *J*

- Constraint

  y position of shear center *ey*≧10(mm)

  Second moment of Inertia *Iy*≧900,000 (mm^4)

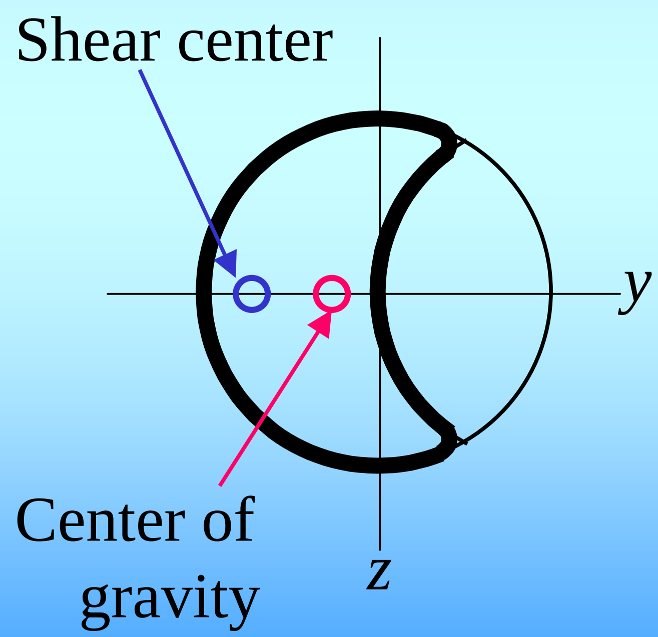
  Second moment of Inertia *Iz*≧100,000 (mm^4)



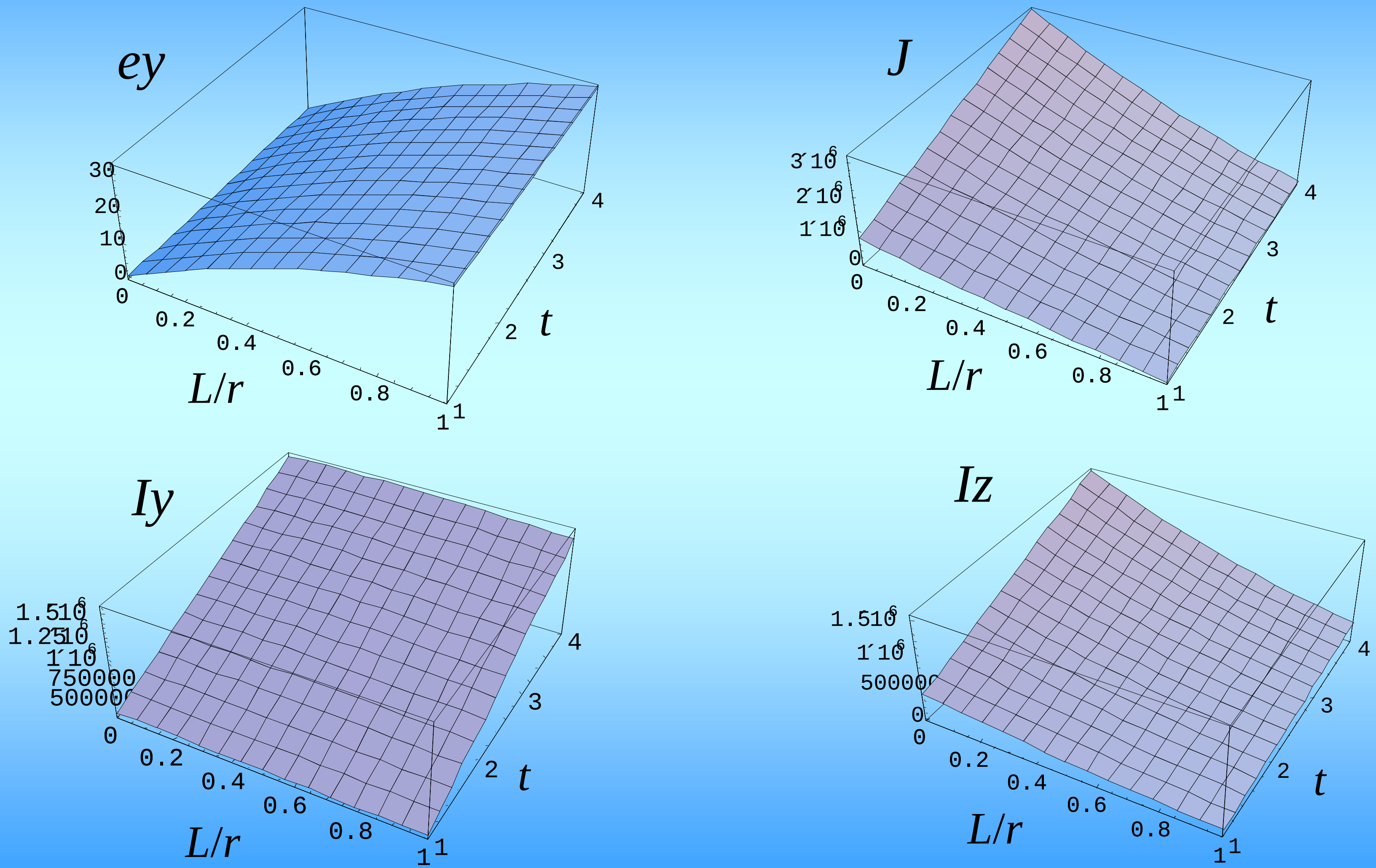
Section of beam

Parametric Studies Based on DOE

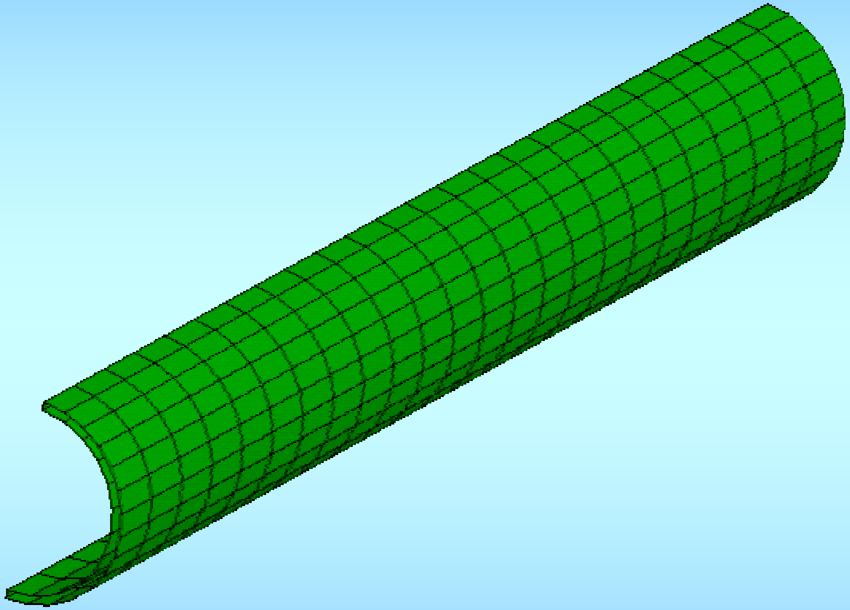
- Two factors, three levels in DOE
  - L/r = 0.0,0.5,1.0 where r=50(mm)
  - t=1.0,2.5,4.0 (mm)
- Evaluation function
  - y position of shear center ey
  - Second moment of Inertia Iy,Iz
  - Polar moment of Inertia J

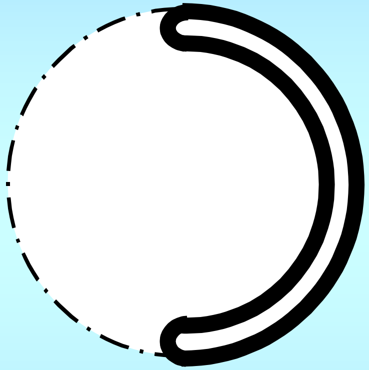


Response surface



Optimal design





Optimal Solution L/r=1.0, t=2.47(mm)

Response Surface Methodology
and
Its application to automotive suspension designs

Thank you very much !