Final Examination

MEAM305 Introduction to Finite Element Methods

April 21, 98W

1. Knowing the parametric coordinates of the node of the HEXA8 element as follows (20 points) :

node	ξ	η	ζ
1	-1	-1	-1
2	+1	-1	-1
3	+1	+1	-1
4	-1	+1	-1
5	-1	-1	+1
6	+1	-1	+1
7	+1	+1	+1
8	-1	+1	+1

(a) define the shape function $N_3(\xi,\eta,\zeta)$ of node 3, and (b) evaluate it at node 2, at the centroid, and at a point $(\xi,\eta,\zeta)=\left(-\frac{1}{\sqrt{3}},-\frac{1}{\sqrt{3}},+\frac{1}{\sqrt{3}}\right)$. (c) Sketch the function profile of the shape function N_3 on the surface defined by $\eta=0$ and $\zeta=0$. It should become a function of ξ in the interval (-1,1).

2. When the HEXA8 element is used to approximate the geometry and displacement, and when the strain assumed approximation is considered, explain why the component ε_x of the strain is approximated by a polynomial

$$\varepsilon_x = a_0 + a_1\eta + a_2\zeta$$

where a_0, a_1, and a_2 are unknown coefficients. Similarly, the shear strain γ_{xy} should be approximated by

$$\gamma_{xy} = b_0 + b_1\zeta$$

in the strain assumed element. Explain why this approximation makes sense. Here we have assumed that the local coordinates (x,y,z) are almost parallel to the parametric coordinates (ξ,η,ζ). (20 points)

3. Suppose that a given function

$$g(\xi)=1-\xi^2$$

in the interval $(-1,1)$, must be approximated by a constant function

$$g_{approximation}(\xi) = a,$$

where the coefficient *a* is an appropriate number. Find *a* by using the least squares method. (20 points)

4. For the finite element model consisting of two bar elements axially loaded as shown in Fig. 1. Let Young's modulus be E1 and E2, let the length of the elements be L1 and L2, and let the cross sectional area of the elements be A1 and A2, respectively. (20 points)

(1) Find the sensitivity of the displacement at the loading point, that is, at node 3, if design variables is the cross sectional area A1.

(2) Find the sensitivity of the axial stress of the first bar element when a design variable is Young's modulus E1 of the second bar element.

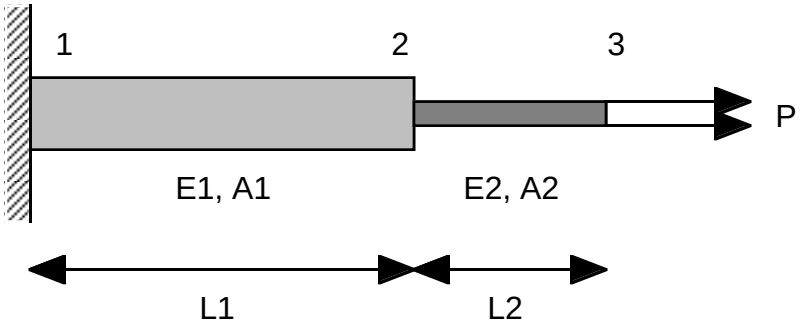

Figure 1 Two Bar Element Model (Axially Loaded)

5. How is the stress gradient related to the finite element approximation error ? Similarly how is the size of the finite element related to the finite element approximation error ? (10 points)

6. Explain the h-element and p-element in the adaptive finite element method. (10 points)