**Differentiation of an integral function**

We shall describe a differentiation of a special integral function $\int_0^{\phi(t)} \psi(t,\tau)\,d\tau$ with respect to time *t*:

$$\frac{\partial}{\partial t}\int_0^{\phi(t)} \psi(t,\tau)\,d\tau = \lim_{\Delta t\to 0}\frac{\int_0^{\phi(t+\Delta t)} \psi(t+\Delta t,\tau)\,d\tau - \int_0^{\phi(t)} \psi(t,\tau)\,d\tau}{\Delta t}$$

$$= \lim_{\Delta t\to 0}\frac{\int_0^{\phi(t)+\phi'(t)\Delta t} \psi(t+\Delta t,\tau)\,d\tau - \int_0^{\phi(t)} \psi(t,\tau)\,d\tau}{\Delta t}$$

$$= \lim_{\Delta t\to 0}\frac{\int_{\phi(t)}^{\phi(t)+\phi'(t)\Delta t} \psi(t+\Delta t,\tau)\,d\tau + \int_0^{\phi(t)} \psi(t+\Delta t,\tau)\,d\tau - \int_0^{\phi(t)} \psi(t,\tau)\,d\tau}{\Delta t}$$

$$= \lim_{\Delta t\to 0}\frac{\phi(t)+\phi'(t)\Delta t-\phi(t)}{\Delta t}\psi\left(t+\Delta t,\phi(t)\right) + \lim_{\Delta t\to 0}\int_0^{\phi(t)}\frac{\psi(t+\Delta t,\tau)-\psi(t,\tau)}{\Delta t}\,d\tau$$

$$= \phi'(t)\psi\left(t,\phi(t)\right) + \int_0^{\phi(t)}\frac{\partial\psi(t,\tau)}{\partial t}\,d\tau$$

that is

$$\frac{\partial}{\partial t}\int_0^{\phi(t)} \psi(t,\tau)\,d\tau = \phi'(t)\psi\left(t,\phi(t)\right) + \int_0^{\phi(t)}\frac{\partial\psi(t,\tau)}{\partial t}\,d\tau$$