Review Problems for the Final Examination
MEAM 501 Analytical Methods in Mechanical Engineering

1997 Fall

1. State the following definitions, properties, and/or concept :

(1) What is the Lagrange interpolation of a function f defined on an interval (a,b).

(2) What is the Legendre polynomilas defined on the interval (-1,1) ?

(3) What is the Chebyshev polynomilas defined on the interval (-1,1) ?

(4) What is the Hermite polynomilas defined on the interval $(-\infty,\infty)$?

(5) What is the Lagerre polynomilas defined on the interval $(0,\infty)$?

(6) Obtain the 2 point Gauss-Legendre quadrature to integrate a function defined on the interval (-1,1), that is, obtain the quadrature points and the associate weights.

(7) Obtain the 3 point Gauss-Lagerre quadrature to integrate a function defined on the interval $(0,\infty)$, that is, find the 3 point quadrature points and associated weights.

(8) Integrate by using a numerical method to integrate

$$\int_{-\infty}^{+\infty} \frac{\exp(-x^2)\sin x}{1+x^2}\,dx$$

with the accuracy of 10^{-6}.

(9) What is the cubic Hermite interpolation of a function f defined on the interval (0,1) ?

(10) What is the Bezier spline approximation of a curve ?

(11) What is the B-spline approximation of a curve ?

(12) What is the minimum principle ?

(13) What is the trapezoidal rule ?

(14) What is the Simpson rule ?

(15) What is the exponential transformation for quadrature ?

2. Obtain the first variation of the following functionals, the necessary conditions, and Euler's equations on the admissible set K :

(1) $$J(v) = \frac{1}{2}\int_0^1 \left\{ (v')^2 + xv^2 \right\} dx - \int_0^1 fv\,dx$$

$$K = \left\{ v \in V : v(0) = v(1) = 0 \right\}$$

$$V = \left\{ v : \text{piecewise continuously differentiable functions on } (0,1) \right\}$$

(2) $J(v)=\frac{1}{2}\int_0^1\left\{(v')^2+xv^2\right\}dx+\frac{1}{2}k_0v(0)^2-\int_0^1 fvdx-Pv(0)$

$K=V$

(3) $J(v)=\frac{1}{2}\int_0^1\left\{\left(EI(x)v''\right)^2-P(v')^2+k(x)v^2\right\}dx+\frac{1}{2}k_0v(0)^2+\frac{1}{2}k_1v'(0)^2$

$$-\int_0^1 fvdx-Fv(0)-Tv'(0)$$

$K=V$

$V=\{v$: piecewise *twice* continuously differentiable functions on $(0,1)\}$

3 Solve the minimization problem by the Ritz method :

$$\min_{\substack{v \\ \text{such that} \\ v(0)=0}} J(v)$$

where

$$J(v)=\frac{1}{2}\int_0^1\left\{(v')^2+xv^2\right\}dx-\int_0^1 2vdx.$$

4. Consider a curve defined by

$$\begin{cases} x=\cos\left(\theta+\theta^2\right) \\ y=\sin(\theta)+\sin\left(\theta^2\right) \\ z=\theta/2\pi \end{cases}$$

using a parameter θ such that $\theta\in(0,2\pi)$.

(1) Obtain the expression of the tangent vector ***t***.

(2) Obtain the normal and bi-normal vectors ***n*** and ***b***, respectively.

(3) What is the total length of this curve ?