Homework #4
AM 502 Differential Equation Methods in Mechanics

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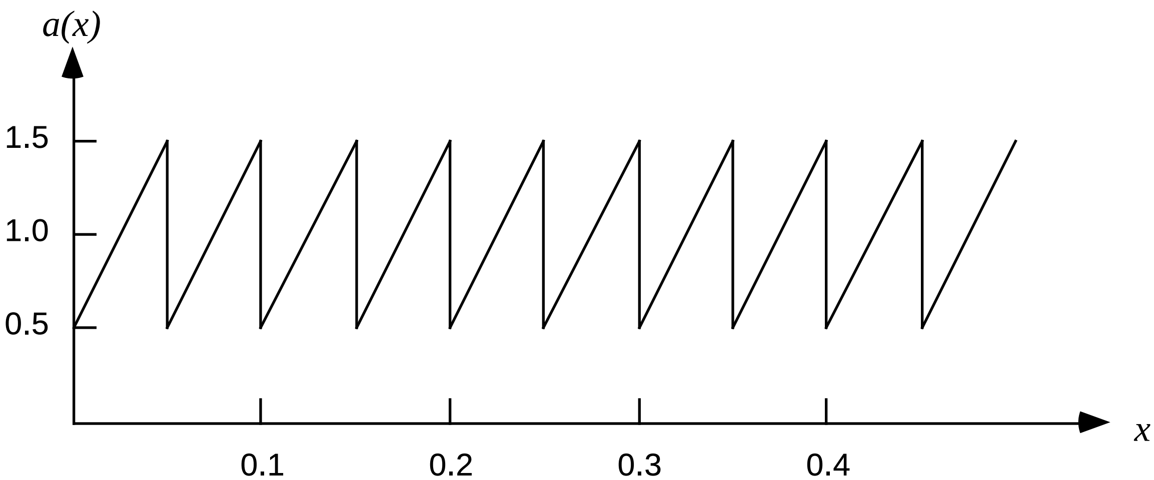
1. Consider an oscillatory coefficient shown in below



and the functional

$$F(u) = \frac{1}{2}\int_0^1 \left\{ a(x)\left(\frac{du}{dx}\right)^2 + \sin(2\pi x)u^2 \right\} dx - \int_0^1 \left(1 + x^2\right) u\,dx$$

for the minimization problem

$$\min_u F(u)$$

among the admissible functions such that $u(0) = u(1) = 0$.

(1) Find the first variation δF of the functional F and its Euler's equation by setting δF = 0 for every δu = 0.

(2) Find the homogenized coefficient a^H and homogenized functional $F^H\left(u^H\right)$, and then solve the homogenized problem to find u^H that minimizes the homogenized functional $F^H\left(u^H\right)$. Furthermore, estimate the maximum difference of $u^H - u$ and $\frac{d}{dx}\left(u - u^H\right)$ based on the homogenization asymptotic expansion.

(3) Solve the original problem by FEM or FDM, and compare these results with the one obtained by the homogenization method.

2. Let us consider the total potential energy for a beam bending problem

$$F\left(v^{\varepsilon}\right)=\frac{1}{2}\int_{0}^{L} EI^{\varepsilon}\left(\frac{d^{2}v}{dx^{2}}\right)^{2} dx-\int_{0}^{L} b^{\varepsilon} v dx-\int_{0}^{L} m^{\varepsilon}\frac{dv}{dx} dx$$

where E is Young's modulus, I^{ε} is the moment of inertia depending on the microstructure, b^{ε} is the applied transverse force, m^{ε} is the applied distributed moment, and L is the length of the beam. Here the beam is reinforced by uniformly placed ribs, say, the original beam has b x h rectangular cross section and it is reinforced by $\varepsilon/2$ x $2h$ ribs in the both sides. Assume that the total volume of the ribs is $2bhL$.

(1) For the asymptotic expansion

$$v^{\varepsilon}(x)=v_{0}(x)+\varepsilon v_{1}(x,y)+\varepsilon^{2} v_{2}(x,y) \qquad with \qquad y=\frac{x}{\varepsilon}$$

described in above, define the microstructure Y.

(2) Obtain the homogenized moment of inertia I^{H}, homogenized distributed transverse force b^{H}, and homogenized distributed moment m^{H}.

(3) Derive Euler's equation and the natural boundary condition(s) for u_0 if the minimum principle of F is considered in

$$V=\left\{v \quad H^{2}(0,L) \,\middle|\, v=\partial v=0 \quad at \quad x=0\right\}$$

where $H^{2}(0,L)$ is the sobolev space for the beam bending problems :

$$H^{2}(0,L)=\left\{v \quad L^{2}(0,L) \,\middle|\, \partial v \quad L^{2}(0,L) \quad and \quad \partial^{2} v \quad L^{2}(0,L)\right\}.$$