IOE 510 HW #9 Winter 2002 Due. 27. Mar.

#1.

4.30 For the LP given at the top of page 223 check whether $x = (7, 0, \frac{5}{2}, 0, 3, 0, \frac{1}{2})^T$ is an optimum feasible solution.

$$
\begin{array}{ll}
\text{Maximize} & z(x) = x_1 + 2x_2 + x_3 - 3x_4 + x_5 + x_6 - x_7 \\
\text{Subject to} & x_1 + x_2 - x_4 + 2x_6 - 2x_7 \leqq 6 \\
& x_2 - x_4 + x_5 - 2x_6 + 2x_7 \leqq 4 \\
& x_2 + x_3 + x_6 - x_7 \leqq 2 \\
& x_2 - x_4 - x_6 + x_7 \leqq 1 \\
& x_j \geqq 0 \quad \text{for all } j
\end{array}
$$

#2

Minimize

$$z(x) = \sum\sum c_{ij}x_{ij}$$

Subject to

$$
\begin{array}{lll}
\sum_j x_{ij} = a_i & i = 1 \text{ to } m & \\
\sum_i x_{ij} = b_j & j = 1 \text{ to } n & (4.10) \\
x_{ij} \geqq 0 & \text{for all } i, j &
\end{array}
$$

4.31 For the transportation problem (4.10) with the following data, find an optimum feasible solution using the complementary slackness conditions, given that $u = (u_i) = (0, 3, -4)$, $v = (v_j) = (7, 2, -5, 7)$ is an optimum dual solution.

$$c = (c_{ij}) = \begin{pmatrix} 7 & 2 & -2 & 8 \\ 19 & 5 & -2 & 12 \\ 5 & 7.2 & -9 & 3 \end{pmatrix}$$

$a = (a_i) = (3, 3, 4)$, $b = (b_j) = (2, 3, 2, 3)$

#3

4.35 For the LP given below compute the primal and dual solutions associated with the basis corresponding to the basic vector (x_3, x_5, x_1). Is it an optimum basis? Characterize the set of optimum solutions of this problem using the complementary slackness theorem. Do the same for the basic vector (x_5, x_2, x_3, x_8) for the LP given at the top of page 224.

x_1	x_2	x_3	x_4	x_5	x_6	x_7	$-z$	b
1	4	1	−2	2	5	1	0	11
0	−2	0	2	−1	−1	−1	0	−4
1	1	0	−1	1	3	2	0	4
2	7	1	−1	2	13	2	1	0

$x_j \geqq 0$ for all j; minimize z.

$$
\begin{array}{lll}
\text{Minimize} & 3x_1 - 2x_2 + x_3 - x_4 - 5x_5 + 4x_6 - 2x_7 - 3x_8 & \\
\text{Subject to} & x_1 + x_2 + x_3 + x_4 + x_5 + x_6 + x_7 + x_8 & = 14 \\
& x_1 + x_2 + x_3 \qquad\quad + x_5 + x_6 + x_7 & = 9 \\
& x_1 + x_2 \qquad\qquad\quad\; + x_5 + x_6 & = 5 \\
& x_1 \qquad\qquad\qquad\qquad\; + x_5 & = 2 \\
& x_j \geqq 0 \quad \text{for all } j &
\end{array}
$$

#4

4.14 Using Farkas' lemma, prove the following theorem of the alternative. If A is an $m \times n$ matrix and $b \in \mathbf{R}^m$, exactly one of the following two systems has a solution.

Either $\quad Ax = b \qquad\qquad\qquad$ (I)

or $\quad \pi A = 0 \quad \pi b = 1 \qquad$ (II)