

Power Requirements of Swimming: Do New Methods Resolve Old Questions?¹

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SYNOPSIS. A recurring question in the study of fish biomechanics and energetics is the mechanical power required for tail-swimming at the high speeds seen among aquatic vertebrates. The quest for answers has been driven by conceptual advances in fluid dynamics, starting with ideas on the boundary layer and drag initiated by Prandtl, and in measurement techniques starting with force balances focussing on drag and thrust. Drag (=thrust) from measurements on physical models, carcasses, kinematics as inputs to hydro-mechanical models, and physiological power sources vary from less than that expected for an equivalent rigid reference to over an order of magnitude greater. Estimates of drag and thrust using recent advances largely made possible by increased computing power have not resolved the discrepancy. Sources of drag and thrust are not separable in axial undulatory self propulsion, are open to interpretation and Froude efficiency is zero. Wakes are not easily interpreted, especially for thrust evaluation. We suggest the best measures of swimming performance are velocity and power consumption for which 2D inviscid simulations can give realistic predictions. Steady swimming power is several times that required for towing an equivalent flat plate at the same speed.

INTRODUCTION

Humans have long been impressed with the swimming speed and agility of fish and cetaceans. Not surprisingly, this has led many to ask: How do these animals swim? How much does it cost? How efficient are animal swimmers? How do costs and efficiency compare with human vehicles? Such questions have influenced research on vertebrate swimming for centuries, and are now being addressed by new techniques (Table 1), made possible primarily by increased computational power. Here we review approaches to determine the costs of swimming, primarily for steady axial self-propulsion, and how well new insights answer old questions.

The foundation of biomechanical studies of fish locomotion is that the drag and hence thrust can be distinguished in steady swimming. This approach has been developed by biologists (*e.g.*, Gray, 1936) and mathematicians and hydrodynamicist (*e.g.*, Lighthill, 1975). Here we challenge this foundation for axial, undulatory self-propulsion.

THE 20TH CENTURY FOCUS—GUESSING DRAG

Theoretical drag

Historically, research on swimming has taken its lead from engineering and applied mathematics. Solving the Navier-Stokes equation for swimming animals was used for small, worm-like or anguilliform swimmers (Carling *et al.*, 1998). For most fishes, Prandtl's (1904) boundary layer theory and ideas stemming from this seemed more applicable, and focused issues of swimming costs and efficiency on drag.

We define theoretical drag (Webb, 1975) as that of a rigid body, otherwise morphologically equivalent to an animal of interest. A simpler commonly used reference is the theoretical drag for a flat plate with equivalent surface area and aspect ratio. As with all measures of drag and thrust, values have been compared as the drag (equal to thrust) force, power, or as a coefficient (Fig. 1).

Originally, theoretical drag provided a best estimate for swimming drag, and was central in the especially influential analysis by Gray (1936) of swimming dolphin. Using reported estimates of the swimming speed of a dolphin, the theoretical drag power was substantially larger than the then best estimates of muscle power. The mismatch, dubbed "Gray's Paradox," triggered studies to determine swimming drag and a search for drag-reducing mechanisms. The best estimate of muscle power used by Gray proved to be for a sustained activity, which he equated to power needs in a sprint activity (Bainbridge, 1961). Although apparent inequalities between drag and muscle power were largely resolved by better data on speeds and muscle performance (Bainbridge, 1961), Gray's Paradox continues to be influential, stimulating searches for ways that fish might perform better than human vehicles.

Dead drag

We use the term "dead drag" to refer to all measurements of drag on fresh or preserved carcasses and physical models (Table 1). The latter may be coated with mucus or sheathed in materials intended to mimic surface compliance of the integument.

Results are extremely variable and dead drag measurements vary from slightly smaller than theoretical drag to approximately 10 times greater (Fig. 1). Major contributors to this increased drag are appendages and/or bodies acting like flags (Webb, 1975). Values close to reference drag, sometimes a little below, occur at

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TABLE 1. Summary of approaches used to evaluate flow features and/or forces during swimming.*

Reference (Theoretical) drag	Primarily empirical relationships for rigid bodies and flat plates	Various	Assumes swimming drag is unaffected by body movements. Assumes no drag-reduction other than from body shape.
“Dead” drag	Measured drag of dead, anesthetized, and model fishes	Various, Drucker and Lauder (1999)	Assumes swimming drag is unaffected by body movements.
Direct measurement of forces	Work against loads, load cells, measurement of pressure on body/tail, and increasing viscosity	Various, Webber <i>et al.</i> , (2001)	Various methods measure surplus work, change working relationships between propulsive wave and the water, add destabilizing torques.
Kinematics-based models	Elongated slender body	Various, Pedley and Hill (1999)	Reaction forces assumed dominant. Usually small amplitude motions, sometimes with large amplitude corrections. Swimming kinematics and u are parameters making independent validation difficult.
	Propulsor-element models	Various, Westneat (1996), Walker and Westneat (1997, 2000), Coombes and Daniel (2001)	Thrust determined for lift, drag, and/or acceleration reaction. Many are quasi-static, assuming forces at any instant are equivalent to those under dynamically similar steady state. More recent models place greater emphasis on unsteady forces. Mainly used for appendicular propulsion, occasionally axial propulsion.
Muscle force and power Metabolic rate		Various	Usually assumes all muscle activated simultaneously at limits of performance.
		Various	Must make assumptions for metabolic and muscle efficiency.
Flow visualization	Dyes	Various	Dyes, other markers, tripping wires, Schlieren, and shadowgraphs indicate near-body and far-body flows.
	Particle Image Velocimetry (PIV) and Particle Tracking Velocimetry (PTV)	Stamhuis and Videler (1995), Videler <i>et al.</i> (1999), Müller <i>et al.</i> (1997, 2000, 2001), Lauder (2000), Drucker and Lauder (2001a,b)	PIV has become method of choice. Wake and near-body flow patterns provide clues of flow and momentum changes of body/propulsors. Fine scale observations show boundary-layer flow. Cannot determine thrust for axial self propulsion.
Computational Fluid Dynamics (CFD)		Schultz <i>et al.</i> (1991), Fauci (1996), Liu <i>et al.</i> (1996, 1997), Carling <i>et al.</i> (1998)	Usually models vorticity and pressure distribution near a swimming fish. Trajectories predictable from kinematics provide for independent validation.
Vortex Interaction (Manipulation) Models		Various, Ahlborn <i>et al.</i> (1991, 1997), Triantafyllou and Triantafyllou (1995), Anderson <i>et al.</i> (1998), Wolfgang <i>et al.</i> (1999), Triantafyllou <i>et al.</i> (2000)	Currently combines various components: experiments on highly simplified systems, PIV, computational methods, hydrodynamic modeling.

* Exemplary sources are cited for developments since the mid 1990s. Prior studies are only noted as “various.” These are reviewed by Webb (1975), Blake (1983) and Videler (1993), who have discussed the practical and conceptual problems with various methods, and summarized earlier results. These works should be consulted for original citations of work prior to the early 1990s.

low speeds (Drucker and Lauder, 1999) or for fish with stiff bodies (Q. Bone in Webb, 1975; Blake, 1983). Observations on stiffened fish and rigid models defeat one object of drag measurement of trying to capture possible drag-reducing effects of the integument or shape (Webb, 1975; Blake, 1983; Videler, 1993). In practice, such issues are largely moot for axial undulatory self-propulsion because swimming motions modify flow compared to that for a rigid body.

Theoretical and dead drag (without flag effects) measurements may be applicable for fish swimming with the body held straight and propelled with short base-length oscillatory appendages.

Flow patterns

Visualizing the near-body and boundary-layer was also attempted to provide clues to the potential magnitude of drag (Table 1). Early approaches used flow

markers such as dyes to determine if the boundary layer separated (Aleyev, 1977). Results proved equivocal, some suggesting separation occurred (Aleyev, 1977), while others did not (Webb, 1975; Blake, 1983). Observations of wake flow show a reverse Kármán vortex street, definitive of thrust-production by an axial undulatory self-propulsor. Rosen (1959) suggested that vortices generated by the anterior of the body might act as pegs that a fish could push against resulting in virtually no drag.

Kinematic models for thrust

Problems of measuring drag and the lack of good methods to visualize the flow made it attractive to combine easily measured kinematic data with hydrodynamic models to estimate thrust (Table 1). Small-amplitude elongated-body models have proven especially popular. First, in a bulk-momentum form, only three parameters

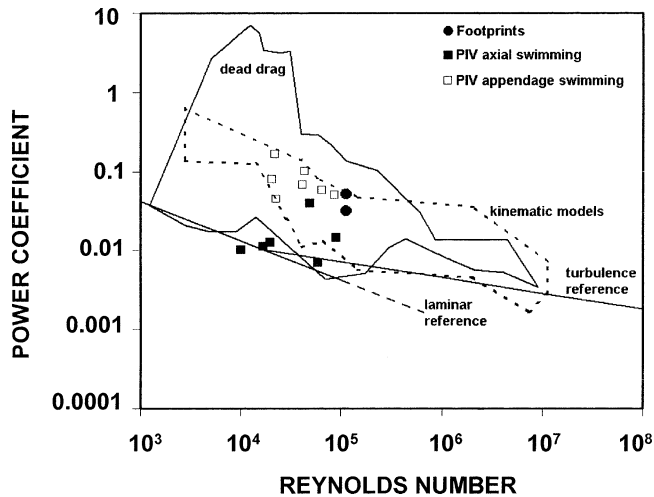


FIG. 1. Comparison of results from several approaches used to determine drag (=thrust). Drag is converted to drag coefficients, expressed relative to Reynolds Number, Re , based on total length. Reference values are shown for the drag coefficient on a flat plate with a fully laminar or fully turbulent boundary layer. Data for theoretical and dead drag, and solutions to kinematic models, are taken from Webb (1975), Blake (1983), and Videler (1993). Data for footprints, a vortex interaction model, are taken from Ahlborn *et al.* (1991). Values for PIV studies are calculated from Drucker and Lauder (1999, 2000, 2001a), Anderson *et al.* (2001), and Müller *et al.* (2001).

characterizing propulsor motions are needed to determine power at a known swimming speed: tail-beat frequency, trailing-edge amplitude and the backward speed of the propulsive wave. Second, the model can be used for many fast swimmers, although lifting-wing models were developed for the fastest swimming fish with high aspect ratio oscillatory propulsors (Lighthill, 1975; Blake, 1983; Videler, 1993).

Several numerical models also have been developed to estimate thrust. These are often quasi-static in form, assuming the force on a propulsor at some instant is equal to that in steady motion. An early example is Taylor's (1952) resistive model for anguilliform propulsion. Blake (1983) used numerical models extensively to examine paired fin propulsion. The same approach is still used, and increasing computational power allows for more complex motions of more elements and better inclusion of acceleration reaction (Walker and Westneat, 2000).

In general, estimates of mechanical power from hydrodynamic models suggest that if a self-propelled fish could be treated like a submarine, its drag would be 3 to 5 times greater than theoretical drag (Fig. 1).

Muscles and metabolism

The popular kinematics models used for self-propelled axial swimmers predict rates of working and efficiency for the propeller function (Lighthill, 1975). However, the only pathway towards validation of this approach is to compare power output with "shaft power," the input power to the propeller from the muscles and metabolism.

In general, shaft power calculated from fast glyco-

lytic muscle exceeds theoretical drag power (Bainbridge, 1961) as well as the larger power predicted using kinematic models (Webb, 1975; Blake, 1983; Videler, 1993). However, it is difficult to determine muscle shaft power. The myotomal organization of muscle fibers and connective tissues is complex, and possible transmission losses are unknown. In addition, the entire muscle is rarely active simultaneously, and positive work is done in some portions of the myotome and negative work in others (Wakeling *et al.*, 1998; James and Johnston, 1998).

Water tunnels provided another way of estimating shaft power from measured rates of oxygen consumption, converted to mechanical work via oxycaloric equivalents (Blazka *et al.*, 1960; Brett, 1963, 1964). Shaft power is then determined from the product of metabolic and muscle efficiency, neither of which are known with certainty. Together, these are assumed to be about 20%. Drag power is obtained from shaft power using estimates of Froude efficiency (Table 1).

Total metabolism is measured but must be partitioned among various tasks. Usually, standard metabolism (the metabolic rate of an unstressed fish at rest under proscribed conditions) is subtracted from the total metabolism. The resulting net energy is assumed to generate mechanical power to swim. There are many potential pitfalls in this approach. The nature of the statistical model relating metabolic rate and speed affects estimates of standard metabolism (Gordon *et al.*, 1989; Videler, 1993; Webb, 1997). The relationship itself may also be affected by elevated metabolic rates at low speeds due to excitement (Brett, 1964), stability costs (Webb, 1997), anaerobic metabolism supplementing aerobic metabolism at high speeds (Brett, 1964), use of various gaits (Alexander, 1989; Webb, 1994), and shifts from branchial to ram ventilation (Freadman, 1981). In addition, part of the net energy drives maintenance systems. Supplying and removing metabolites, for example, might be a large part of increased metabolism with speed (Jones, 1971).

In spite of these uncertainties, net metabolic rates are consistent with drag power similar to that predicted by hydromechanical models. Thus both metabolic and muscle power suggest drag is several times its theoretical value (Alexander, 1967; Webb, 1975).

LATE 20TH CENTURY DEVELOPMENTS

Estimates of drag or thrust leading into the 1990s ranged from about zero to 10 times theoretical drag (Fig. 1). Increased computing power has greatly expanded abilities for flow visualization and modeling (Table 1), methods which have been used to reconsider the earlier questions of the magnitude of drag.

Particle Image Velocimetry (PIV)

Particle tracking velocimetry (PTV) and particle image velocimetry (PIV) combine laser illumination, digital imaging, and computer analysis of particle trajectories to explore flow from the boundary-layer to the wake. PIV is the method of choice, but both PTV and

PIV are conceptually the same (Stamhuis and Videler, 1995). Modern PIV systems seed water at high densities, automatically track particles moving short distances and use various algorithms to determine water velocities. Currently, 2D systems are commonly used, with the 3D flow being inferred from 2D slices at different positions in assumed identical motions, observed at the same phase but at different times. This assumption will be removed as 3D systems are deployed.

PIV analysis of boundary layer flow (Anderson *et al.*, 2001) shows increased shear rates of 3.6 for swimming smooth dogfish (*Mustelus canis*) and 1.5 to 1.9 for scup (*Stenotomus chrysops*) compared to a rigid reference. Analysis of reconstructed vortex rings (Lauder, 2000) has been used to estimate thrust (Müller *et al.*, 1997, 2001; Drucker and Lauder, 1999, 2000, 2001a, b). Rates of working from these calculations range from less than that required to overcome theoretical drag to over 5 times that amount (Fig. 1).

Computational Fluid Dynamics (CFD)

The descendant of kinematics-thrust analytical modeling is computational fluid dynamics. Early viscous applications (Liu *et al.*, 1996, 1997) claimed rates of working (for modest Reynolds numbers, Re , of swimming tadpoles) that were four times theoretical drag and a Froude efficiency for the propeller of 80%. This important early work appears to include some uncertainties. For example, boundary layers appear somewhat thick, and velocities within them include unexpected speed maxima (*e.g.*, Fig. 3B in Liu *et al.*, 1996). Bodies that would be expected to accelerate on the basis of the model results may not do so (*e.g.*, Fig. 5 in Liu *et al.*, 1996).

Inviscid CFD models (for Re above 100) have also been developed that apply to all but the most viscous swimmers. Here the boundary layer is initially ignored, but the result can be modified after the fact to include viscous effects. Because of the nature of inviscid flow, these techniques are often called *vortex methods* because point vorticity can be used to satisfy a required non-penetrating boundary condition at the fish surface. The fluid is *irrotational* at all points, except next to a vortex street wake behind the fish. As first used by Schultz *et al.* (1991), the pressure distribution is integrated around the body for observed or simulated body movements to find the location and orientation history of the fish. A very similar model was used by Triantafyllou *et al.* (1993) which obtains similar results. Such models use singularities in or on the surface of the fish. They then release vorticity behind the fish at the trailing edge in a Kutta condition that satisfies the Kelvin Theorem requiring zero total circulation in the flow.

An important aspect of CFD models is that fish speed does not have to be a parameter as in analytical kinematic models such as Lighthill's (1975). Thus models such as those of Schultz *et al.* (1991) and Triantafyllou *et al.* (1993) determine speed and acceleration from observed body motions. As a result, models can

be tested with data. In addition, the CFD approach can lead to appropriate 2D models addressing key issues of how fish swim and how much energy is required. We illustrate results from such a modeling analysis. Carangiform swimmers, with a wavelength approximately equal to body length, can achieve highest carangiform speeds (Fig. 2a); the net thrust for a swimming fish is zero (see below), but power consumption is non-zero. The latter is also predicted to be largest for carangiform swimmers (Fig. 2b). When, transport economy is considered, defined as velocity achieved per unit power, anguilliform and thunniform swimmers achieve greater values (Fig. 2c). Power can also be expressed in coefficient form, when minimum values are approximately three times that for towing a flat plate at Re of 10^4 (Fig. 2d). This power consumption is consistent with expectations for physiological power available (Webb, 1975; Blake, 1983; Videler, 1993). Body thickness has a large effect on all these measures of performance. Highest speeds are predicted for a tail beat amplitude of $\pm 10\%$ of the body length (Fig. 3), as common in fishes (Webb, 1975; Videler, 1993).

Vortex-interaction models

One of the most interesting developments in the last decade has been the resurrection of ideas of Rosen (1959) that a vortex is produced upstream of the tail during swimming, usually near the head and this reduces "drag" below that of theoretical drag (Triantafyllou and Triantafyllou, 1995). These modern forms combine PIV, CFD and observations on real fishes (Table 1).

One approach has described "protovortices" on the anterior of the body (Müller *et al.*, 1999, 2000, 2001). Here there appears to be some confusion associated with an adequate definition of a vortex. Vorticity is well defined mathematically as the curl of the velocity. A vortex is a region, or blob, where the vorticity is high. If the vorticity is smeared along a line, as it is in a boundary layer, it can be called a vortex sheet, but not a vortex since large circular motion would not occur. Prandtl Boundary-Layer Theory requires vorticity generation only in a thin boundary layer or at separation, and the Bernoulli equation need not apply.

A different approach was described by Anderson *et al.* (1998) for 2D oscillating foils, with a leading edge or dynamic stall vortex being amalgamated with the trailing edge vortex, resulting in improved thrust and efficiency. They claim Froude efficiency as high as 87% was obtained. However, as detailed below, basic principles lead to the conclusion that Froude Efficiency must be zero so it is not clear how they define Froude efficiency.

Ahlborn *et al.* (1991, 1997) focused on rotational momentum, and argued that this momentum transferred to the water at the start of a tail beat was largely recovered later in the beat cycle. Ahlborn *et al.* (1991, p. 531) report thrust for a typical cruising trout (*Oncorhynchus mykiss*) as half that expected from Lighthill's bulk momentum model, but a rate of working

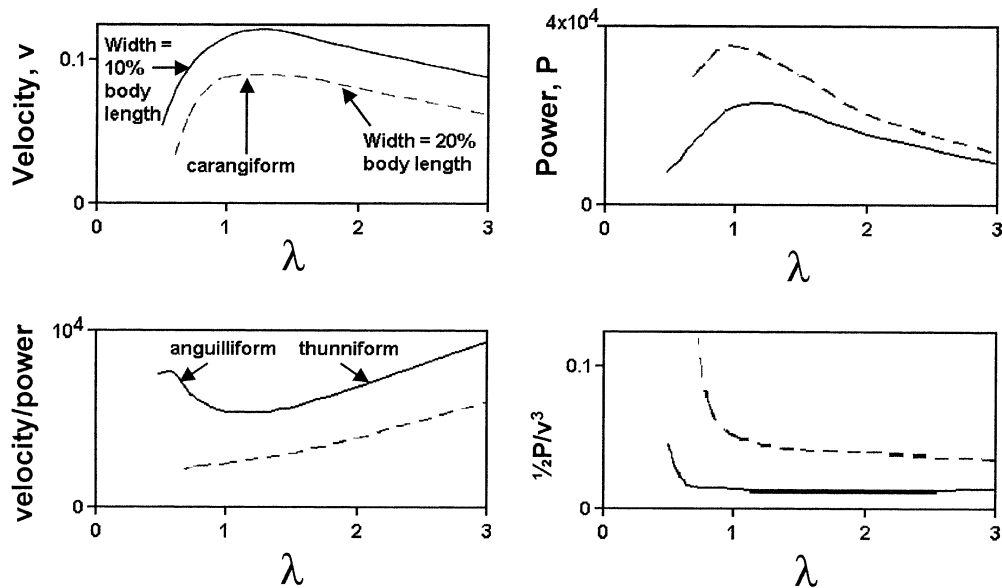


FIG. 2. Exemplary results from a 2D inviscid model for fish swimming (Schultz *et al.*, 1991). The full code is available at <http://www-personal.engin.umich.edu/~schultz/fish/> and for a given fish motion is shown to predict speeds and accelerations according with observations. Four quantities are shown; a) swimming velocity, b) power, c) transport economy, $v/P =$ velocity/power, and d) power coefficient. These are shown as functions of the length of the propulsive wave, (which characteristically increases in amplitude near the tail) in steady swimming. All quantities are dimensionless. Fish length, fluid density, and swimming frequency are set to unity. Hence, λ is the specific wavelength normalized by fish length. Then $\lambda = 1$ indicates one wave along the spine or carangiform-like swimming with increasing amplitude near the tail, $\lambda < 1$ is an anguilliform type wave, and $\lambda > 1$ is more thunniform. In each panel, the solid line is for a maximum width of a typically fish-like fusiform section of 10% of the length, and the dotted line is for a width of 20% body length. a) Dimensionless velocity increases to a maximum at $\lambda \approx 1$, showing that carangiform swimming can achieve the largest velocities. Increased thickness reduces speed achievable for a given motion. Stride length, the distance moved per tail-beat cycle would be slightly higher than the typical value of 0.7 (Videler, 1993). b) Power is closely related to the speed, and gives similar maxima as for speed and the effects of body width. In spite of the absence of viscosity, both the velocity and power are finite for inviscid swimming. c) The transport economy, velocity/power is u-shaped, with carangiform-like swimming being less efficient than anguilliform and thunniform locomotion. However, it should be realized that three-dimensional fluid flows are likely to become important at extremes of small and large λ . d) The power coefficient, C_p , decreases with increasing λ , and is greatly increased at small λ as well as at large thickness. Anguilliform swimmers are typically slender. The lowest C_p is approximately 3 times that for towing a flat plate at $Re = 10^4$.

twice as large. A fish could use vorticity to its advantage if it is produced by an external source, such as the environment or another fish in a school.

Similarly, if large-scale separation is unavoidable upstream of the tail as with fish with a slender caudal peduncle, it is advantageous to have the proper vortex interaction with those lifting surfaces downstream so that energy will not be wasted. Generally, the fish should not produce and use such vortex motion any more than a sailor should advance by blowing on a sail. Yet, Barrett *et al.* (1999) considered vortex-related effects on swimming costs for a tuna-like self-propelling mechanical model. Drag reductions of up to 70% were claimed from comparisons of the net power supplied to the motors of the model *versus* that required to tow the rigid body at the same speed.

However, all these claims are especially confusing in the light of the large body of evidence from other studies (starting with Blazka *et al.*, 1960 and Brett, 1963, 1964) indicating undulatory self-propulsion is energetically expensive. In addition, the postulated role of vorticity production near the head is at variance with inviscid, irrotational models. These models predict trajectories, velocities and power requirements consistent with those observations and biological en-

ergetic expectations, without the vorticity or worrying about how it got produced.

DRAG AND TRUST—TROUBLED SWIMMING STUDIES

Anderson *et al.* (2001) have vividly summed up studies to approximately 1971 as the “troubled field of energetics in undulatory locomotion.” Late 20th century developments give drag and power results that are as variable as earlier studies (Fig. 1). In general, new approaches have not changed from the those which dominated the past century: drag based on some dead or rigid body is estimated and thrust required to overcome that drag is calculated. Depending on the interpretation, the drag during swimming appears to be several times that for the dead/rigid body. This body drag is presumed to be increased by body motions, usually from assumed boundary-layer thinning (Lighthill, 1975). Alternatively, when arguing for reduced drag, drag and thrust are assumed modified by body motions caused by complex vorticity interactions formed near the head.

“Drag reduction” further confuses the issue (Anderson *et al.*, 2001). It was hoped that aquatic vertebrates possessed devices to reduce momentum loss *below that of the best engineered rigid body*. However,

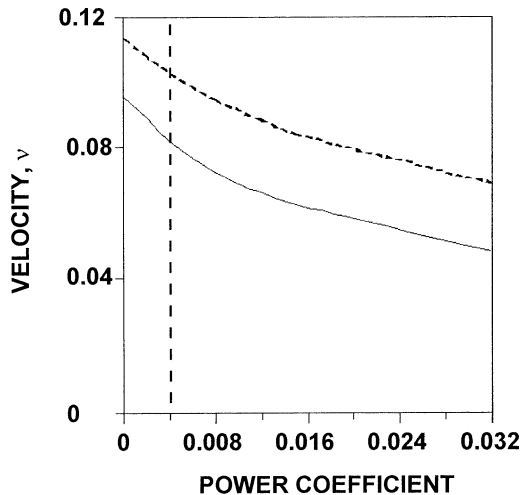


FIG. 3. The effect of artificial viscosity on velocity for a 2D inviscid model for fish swimming (Schultz *et al.*, 1991). Viscosity was modeled as a retarding force on the center of mass. Viscosity equivalent to a drag coefficient of 0.004, the expected equivalent drag on a flat plate, is shown at the vertical line. Adding "viscous" drag reduces velocity as expected, but the expected equivalent drag on a flat plate reduces velocity only by about 10%. There is some numerical viscosity in these types of inviscid CFD model. The number of nodes integrated around the evaluation surface can be varied. Increasing the accuracy of the computation by adding more nodes on the fish surface increases the velocity slightly.

if power output for swimming is high, as suggested by the preponderance of biological data and hydromechanical modeling, *swimming drag reduction can be claimed* relative to this more costly standard (e.g., Bone, 1972; Daniel, 1981; Bechert *et al.*, 1985), while failing to reduce drag relative to that of a best engineered rigid body. Furthermore, given the high rates of working of self-propelling axial undulatory systems, if parasite drag power were indeed small, then power consumed to create that thrust (induced power) must be large. As such, postulated parasite-drag reducing mechanisms would be of marginal importance.

We suggest these conflicts and confusion in studies of fish swimming arise because the drag and thrust cannot be separated and hence have no meaning for axial undulatory self-propelled bodies. Consider a self-propelled ship. Thrust is easily attributed to the propeller and drag to the ship's body. Hence it is reasonable to calculate the thrust and the Froude efficiency for the propeller. However, the thrust and Froude efficiency for the *combination* of the propeller and the ship are both zero at steady speeds. Axial undulatory self-propulsors are such combinations, so that separation of drag and thrust for most swimmers is not possible. Put another way, momentum losses and gains are co-mingled. Unless a fish is trying to "stir up the water," it performs no useful work. Therefore Froude efficiency is zero. A corollary is that evaluation of the fish wake is challenging, but cannot be used to determine drag or thrust: averaged over a tail-beat cycle, the momentum flux in every direction in the wake must sum to zero.

In terms of questions of drag, thrust, and efficiency (rather than the equally important question of how fish swim), we suggest that the interests of a fish and the scientist studying it are primarily concerned with two more practical swimming features for a given motion: what is the swimming speed and what is the power required? Therefore, we suggest understanding of the mechanics and biology of swimming will be facilitated by thinking in terms of power. The basis for comparison among swimmers would then be variables such as "miles per gallon." This is consistent with concepts used in flying, where pilots tend to think in terms of power needs, but may refer to power incorrectly as drag! (Anderson and Eberhardt, 2001). In the end, the power available from the engine is what counts for pilots, and in terms of overall limiting environmental resources of energy and materials, it is important for animals.

We suggest that inviscid CFD models can provide an effective approach for examining factors affecting speed and performance. Surprisingly, even 2D inviscid models only slightly overestimate swimming speed and slightly underestimate swimming power requirements (Schultz *et al.*, 1991; Triantafyllou *et al.*, 1993; Pedley and Hill, 1999). In its simplest form using basic physical concepts, an inviscid model conserves not only angular momentum (required for directional modeling), but also satisfies three conservation laws for mass momentum and energy, respectively:

$$\rho \oint_S \vec{v} \, d\vec{A} = 0$$

$$\rho \oint_S \vec{v} \vec{v} \, d\vec{A} = \vec{F}_s$$

$$\rho \oint_S \left(\vec{u} + \frac{v^2}{2} \right) \vec{v} \, d\vec{A} = \dot{Q} + \dot{W}$$

where ρ = fluid density, v = swimming speed, \vec{u} = specific internal energy (essentially specific heat \times temperature), A = wetted area, F_s = sum of thrust and drag, Q = rate of heat transfer, W = rate of working and S = an evaluation surface bounding a control volume including the system of interest. Other than fluid incompressibility and cyclic averaging, no assumptions are required!

There are typically two choices for S : 1) the outer surface of the fish, or 2) a virtual surface sufficiently far from the fish body that local effects are negligible yet sufficiently close that viscous effects are also negligible. The latter choice is usually used by those analyzing swimming from wake evaluation. Conservation of mass is trivially satisfied for a self-propelling fish since the water entering the forward control surfaces must equal that leaving at the other side.

F_s is the sum of the thrust and drag and hence also is identically equal to zero for steady swimming. Consequently, there will be no momentum surplus or def-

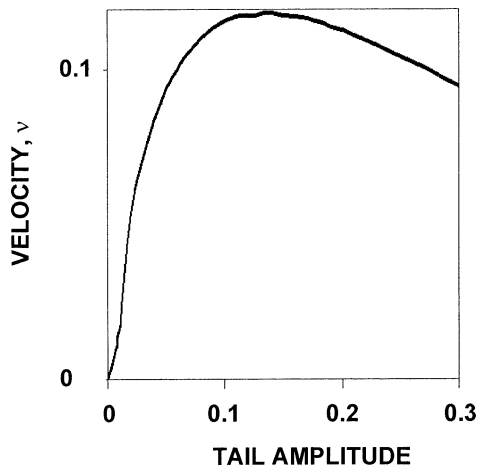


FIG. 4. The effect of tail amplitude on swimming velocity derived from a 2D inviscid model for fish swimming (Schultz *et al.*, 1991). Velocity is maximized when tail amplitude is near 0.1, when the caudal fin motion travels $\pm 10\%$ of its body length. This amplitude is typical of carangiform motion.

icit in the fish wake and no new thrust estimates are possible. However, an analysis of the excess kinetic energy in the wake can lead to an estimate for the power requirement \dot{W} .

Most models of fish swimming take a 2D approach. A 3D analysis would capture and show more detail about fish swimming. Indeed, at first glance, many fish may appear not to be sufficiently flat, without the large aspect ratios necessitating 3D analysis. In practice, body forces are concentrated near a thin caudal fin which has the requisite aspect ratio, lessening the need for 3D analysis. Furthermore, 3D models are very expensive, complicated to program, and tend to be less precise. Similarly, Navier-Stokes computations are typically underresolved (Liu *et al.*, 1996, 1997; Carling *et al.*, 1998) for all but the smallest of swimmers (spermatozoa, larvae as in Fauci 1996). Fortunately, comparison of results from 2D inviscid models with observed performance suggests this approach can be sufficient and provide very useful model-based information (Schultz *et al.*, 1991; Triantafyllou *et al.*, 1993; Pedley and Hill, 1999). We are reminded of Einstein's well-known derivative of Ockham's Razor, that models should be as simple as possible, but no simpler.

Inviscid models, whether 2- or 3-D, by definition simplify the real fluid by neglecting viscosity. Nonetheless, such "zero drag" models give finite speeds (only modestly higher than those seen in experiments or in simulations where appropriate viscous effects are applied) and non-zero power that is consistent with metabolic studies. In addition, if viscosity is artificially added in an inviscid model, it has only a small effect on performance (Fig. 4).

An energetic approach also clarifies the drag controversy. The controversy is a result of the difficulty in decomposition of momentum changes. A possible measure of drag is shear stress, but normal forces for inviscid flow can be viewed as responsible for most

of the power loss. Any sort of decomposition seems not to be meaningful. However, it might be argued that the tail of many fishes is so deep relative to the caudal peduncle that the tail can be treated as a discrete propeller and body drag similarly treated as a separable entity. This might prove to be an acceptable simplification for the most thunniform of swimmers and also many paired-fin swimmers. But for the majority of self-propelled axial undulatory swimmers the degree to which momentum changes are intertwined around the body, caudal peduncle, and tail, continuing to treat drag as a separable force is likely to sustain controversy and confusion.

Nevertheless, an admittedly contrived, yet well-defined "drag" can be obtained from the finite power requirements of swimming. The power required to propel the flat plate or similar passive object is equal to $\dot{W} = F_s v$. Since \dot{W} and v can be predicted, the "drag," F_s can be derived. However, here the "drag" is derived from a rate of working, avoiding the problem of Froude efficiency, which as noted above is zero. Therefore, the meaning of this "drag" is not the same as that used by most researchers through the previous century. It would seem wiser to convert power measures not to "drag" but to non-dimensional power coefficients, numerically of the same order but conceptually different from "drag coefficients."

CONCLUSIONS

We consider the best measures of swimming performance are speed and power consumption. More commonly determined thrust and drag are equal, inseparable, and open to interpretation. Wakes are important but not easily interpreted, especially for thrust evaluation. 2D inviscid simulations can give realistic predictions of swimming speed and power consumption. Independent of viscosity, steady swimming power is several times that required for an equivalent flat plate towed at the same speed.

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