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Technical Note

Thermal convection in a horizontal porous layer with spatially periodic boundary temperatures: small *Ra* flow

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Abstract

This study considers a small Rayleigh number thermal convection in a fluid-saturated porous medium between two infinite-horizontal walls. The lower and upper walls have sinusoidal temperature distributions with a wave number and a phase difference, and the effect of the parameters on the flow and heat transfer characteristics is investigated. For a given wave number, an out-of-phase configuration yields minimum heat transfer at the walls. Maximum heat transfer occurs at the wave number of 2.286 with an in-phase configuration.

1. Introduction

Though the thermal convection in porous media has been studied extensively [1–3], relatively few studied the convection with boundary surfaces having non-uniform temperatures [4–8]. In particular, the study on the system with spatially periodic boundary temperatures is very rare [6–8]. Poulikakos and Bejan [6] and Bradean et al. [7] investigated the convection in a semi-infinite porous medium bounded by a horizontal wall with periodic heating and cooling. Recently, Yoo [8] investigated the convection in a vertical slot with periodic boundary temperatures.

In this note, we consider a steady thermal convection in a fluid saturated porous layer between two infinite-horizontal walls kept at spatially periodic temperatures (Fig. 1). The configuration of the present study is motivated by a system with regularly spaced multiple heat sources such as electronic components and energy storage systems with periodic-spaced piping. The wave number (k) and the phase difference (β) represent the frequency and the stagger of arrangement of the heat sources, respectively. The present configuration is different from the standard Rayleigh–Bénard problem in that we consider the case when there is no mean temperature difference between the lower and the upper walls and no static state without fluid flow. The non-uniform wall temperature generates convective flow that is dependent on the wave number (k) and phase difference (β). We investigate the effect of the configuration (k, β) of the spatial non-uniformities on the flow and heat transfer characteristics.

2. An analytical solution for small *Ra*

We consider a steady state, two-dimensional, thermal convection in a fluid-saturated isotropic porous medium. The dimensionless Darcy–Boussinesq equations [6] and the boundary conditions are given by

$$\nabla^2 \Psi = -Ra \frac{\partial \theta}{\partial x} \tag{1}$$

$$\nabla^2 \theta = \frac{\partial \theta}{\partial x}\frac{\partial \Psi}{\partial y} - \frac{\partial \theta}{\partial y}\frac{\partial \Psi}{\partial x} \tag{2}$$

$$\Psi = 0 \quad \text{at } y = 0, 1 \tag{3}$$

$$\theta = \sin(kx) \quad \text{at } y = 0 \tag{4}$$

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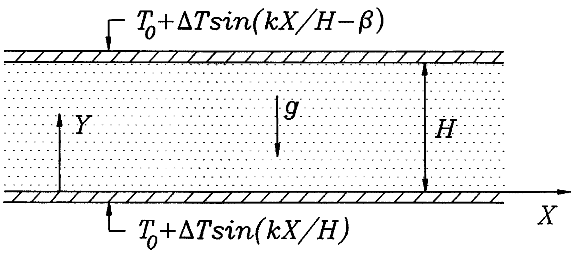

Fig. 1. Problem configuration: a fluid-saturated porous medium is contained between two-infinite horizontal impermeable walls with sinusoidal temperature variations.

$$\theta = \sin(kx - \beta) \quad \text{at } y = 1 \tag{5}$$

where the dimensionless streamfunction (Ψ) and temperature (θ) are defined as $(u, v) = (\partial\Psi/\partial y, -\partial\Psi/\partial x)$ and $\theta = (T - T_0)/\Delta T$, respectively. The Darcy-modified Rayleigh number (Ra) is defined as $Ra = Kg\alpha H\Delta T/\kappa\nu$, where K is the permeability of the porous matrix, α and ν are the thermal expansion coefficient and the kinematic viscosity of the fluid, respectively, κ is the effective thermal diffusivity of the saturated porous medium, H is the thickness of porous layer, and g is the gravitational acceleration.

The governing equations (1)–(5) possess the following point symmetries:

$$\Psi(\beta/k - x, 1 - y) = \Psi(x, y),$$
$$\theta(\beta/k - x, 1 - y) = -\theta(x, y) \tag{6}$$

We define the mean Nusselt number ($\overline{Nu}$) with the averaged heat transfer on the walls over one wavelength $2\pi/k$ ($k \neq 0$):

$$\overline{Nu} = -\frac{k}{2\pi}\int_0^{2\pi/k} \frac{\partial\theta}{\partial y}\,dx \quad \text{at } y = 0, 1 \tag{7}$$

Eqs. (1)–(5) are solved by expanding $\Psi(x, y)$ and $\theta(x, y)$ in powers of Ra [8]:

$$\Psi(x, y) = \Psi_0(x, y) + Ra\Psi_1(x, y) + \cdots,$$
$$\theta(x, y) = \theta_0(x, y) + Ra\theta_1(x, y) + \cdots \tag{8}$$

The solutions to leading order are

$$\Psi_0(x, y) = 0,$$
$$\theta_0(x, y) = f(1 - y)\sin(kx) + f(y)\sin(kx - \beta) \tag{9}$$

$$\Psi_1(x, y) = g(1 - y)\cos(kx) + g(y)\cos(kx - \beta) \tag{10}$$

$$f(y) = \frac{\sinh(ky)}{\sinh(k)},$$
$$g(y) = \frac{\cosh(k)}{2\sinh^2(k)}\sinh(ky) - \frac{y}{2\sinh(k)}\cosh(ky) \tag{11}$$

The zeroth-order solution (Ψ_0, θ_0) represents the pure conduction state without net heat transfer at the walls. The fluid flow in the porous layer can give rise to non-zero averaged heat transfer, and the mean Nusselt number ($\overline{Nu}$) of the first-order temperature distribution is obtained as

$$\overline{Nu} = Ra\,Nu(k, \beta) = Ra[F_1(k) + F_2(k)\cos(\beta)] \tag{12}$$

$$F_1(k) = \frac{-2k^2\cosh(k) + k\sinh(k) + \cosh(k)\sinh^2(k)}{8k\sinh^3(k)} \tag{13}$$

$$F_2(k) = \frac{k^2[1 + \cosh^2(k)] - k\sinh(k)\cosh(k) - \sinh^2(k)}{8k\sinh^3(k)} \tag{14}$$

3. Results and discussion

First, the characteristics of flows for β are shown in Fig. 2 with $k = 3.1$. Fig. 2 presents streamlines and isotherms for several values of β. When $\beta = 0$, upright rectangular-shaped cells appear where the fluid in the plane of $x = \pi/2k$ and $x = 3\pi/2k$ moves upward and downward directions, respectively (Fig. 2(a)). As β increases, the cells are tilted gradually (Fig. 2(b)), and two like-rotating eddies appear in a cell when the tilt from

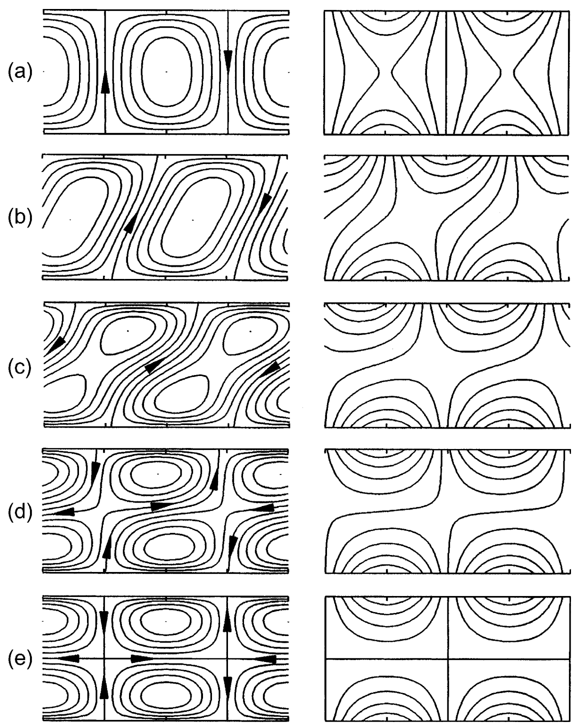

Fig. 2. Streamlines and isotherms of $k = 3.1$ for several phase differences (β): (a) $\beta = 0$; (b) $\beta = 0.5\pi$; (c) $\beta = 0.8\pi$; (d) $\beta = 0.95\pi$; (e) $\beta = \pi$. The range of x is $0 \leqslant x \leqslant 2\pi/k$.

the vertical direction becomes large (Fig. 2(c) and (d)). At $\beta = \pi$, however, upright rectangular-shaped cells appear again by forming two tier-structure cells with two counter-rotating eddies in the vertical direction (Fig. 2(e)). The characteristics of the flow at β^* ($\pi \leqslant \beta^* \leqslant 2\pi$) is identical to that of $\beta = 2\pi - \beta^*$ ($0 \leqslant \beta \leqslant \pi$). Fig. 2(a)–(e) shows a smooth transition of flows from a two-eddy to four-eddy pattern, as β increases from 0 to π. On the other hand, a transition from four-eddy to two-eddy pattern was observed in the vertical slot [8], as β is varied from 0 to π.

The variation of flows with respect to β at small k (<3) is similar to that shown in Fig. 2. At sufficiently large k, however, the thermal interaction between two walls becomes weak, and isolated eddies are formed near each wall, for all β. The flows for $k = 6$ and 12 are presented in Fig. 3 with $\beta = 0$ and $\pi/2$. Fig. 3(a.2) and (b.2) with $k = 12$ show the isolated eddies near the lower and upper walls. When k is large, the fluid in the central part is almost stagnant, and the eddies near each wall are nearly unaffected by each other. The transition of flows with respect to k occurs smoothly: Figs. 2(a) and (b) and Fig. 3 for $\beta = 0$ and $\pi/2$ show a smooth transition from the flow with one eddies to the flow with two eddy in a cell.

The characteristics of the mean Nusselt number ($\overline{Nu}$) for the wave number (k) and the phase difference (β) are investigated with Figs. 4 and 5. Fig. 4 presents the functions $F_1(k)$ and $F_2(k)$ in Eq. (12): $\overline{Nu} = Ra[F_1(k) + F_2(k)\cos(\beta)]$. $F_1(k)$ and $F_2(k)$ have their maximum values at $k_{1m} \approx 2.505$ and $k_{2m} \approx 2.097$, respectively; as k increases, $F_1(k)[F_2(k)]$ increases monotonically at $0 < k < k_{1m} \sim [k_{2m}]$, but decreases at $k > k_{1m} \sim [k_{2m}]$. In particular, $F_2(k)$ approaches zero as k becomes large. The values of $F_2(k)/\mathrm{Max}[F_2(k)]$ at $k = 10$, 12, 15, 18, and 20 are approximately 4×10^{-3}, 6×10^{-4}, 4×10^{-5}, 2×10^{-6}, and 4×10^{-7}, respectively. That is, the averaged heat transfer at the wall becomes independent of the phase difference (β) as the wave number (k) increases ($k \to \infty$), because $Nu(k,\beta) = F_1(k) + F_2(k)\cos(\beta)$ in Eq. (12). And for a given wave number, the maximum and minimum heat transfers occur for in-phase ($\beta = 0$) and out-of-phase ($\beta = \pi$) configurations, respectively, since $F_2(k) > 0$.

The function $Nu(k,\beta)$ for a fixed β is similarly shaped to that for $F_1(k)$ in Fig. 4. And the wave number $k = k_m$ where $Nu(k,\beta)$ has its maximum value is dependent on β. Fig. 5 presents k_m and the corresponding $Nu(k_m,\beta)$ as functions of β. The values of k_m at $\beta = 0$, 0.25π, 0.5π,

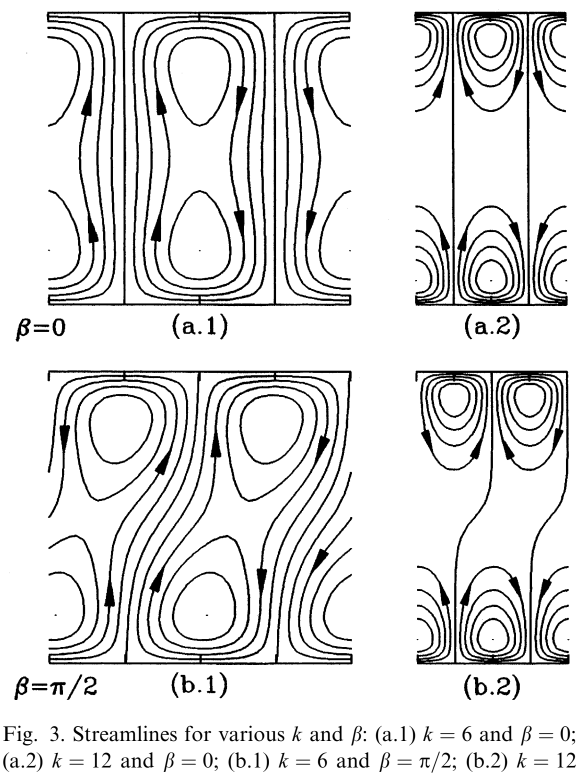

Fig. 3. Streamlines for various k and β: (a.1) $k = 6$ and $\beta = 0$; (a.2) $k = 12$ and $\beta = 0$; (b.1) $k = 6$ and $\beta = \pi/2$; (b.2) $k = 12$ and $\beta = \pi/2$. The range of x is $0 \leqslant x \leqslant 2\pi/k$.

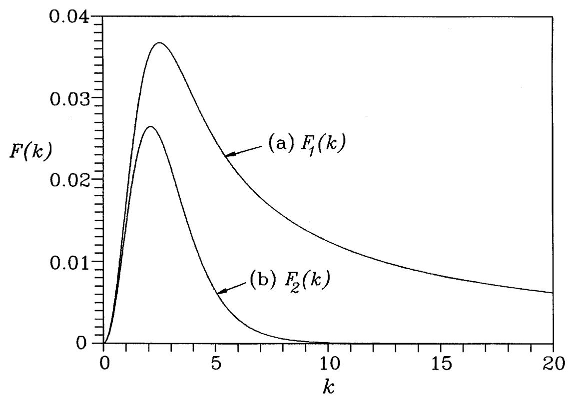

Fig. 4. The functions $F_1(k)$ and $F_2(k)$ showing the dependency of mean Nusselt number on wave number (k): $Nu(k,\beta) = F_1(k) + F_2(k)\cos(\beta)$.

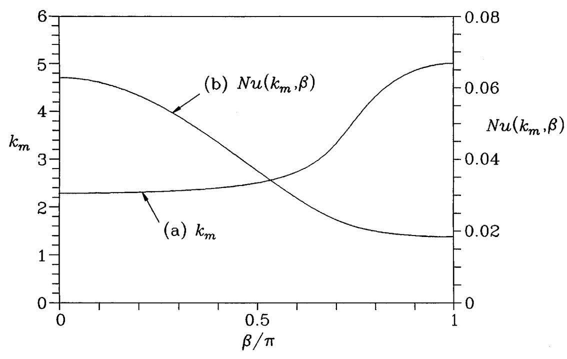

Fig. 5. (a) Wave number (k_m) at which $Nu(k,\beta)$ has its maximum value as a function of β; (b) $Nu(k,\beta)$ at $k = k_m$.

0.75π, and π are $k_m \approx 2.286$, 2.321, 2.505, 3.831, and 5.011, respectively. We can see that maximum heat transfer occurs at $\beta = 0$ and $k \approx 2.286$ (Fig. 5). On the other hand, in the vertical slot with spatially periodic wall temperatures [8], the wave number yielding maximum heat transfer is independent of β, since $Nu(k, \beta)$ is expressed as $Nu(k, \beta) = G(k)\sin(\beta)$; and maximum heat transfer occurs at $k \approx 1.606$ and $\beta = \pi/2$.

In closing, the heat transfer in a horizontal porous layer as a function of phase difference (β) has its maximum and minimum values for in-phase ($\beta = 0$) and out-of-phase ($\beta = \pi$) configurations, respectively; and it becomes independent of the phase difference as the wave number increases ($k \to \infty$). The wave number yielding maximum heat transfer is dependent on the phase difference, and maximum heat transfer occurs at the wave number of $k \approx 2.286$ with in-phase ($\beta = 0$) configuration.

Acknowledgements

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