Monte Carlo Calculation of Point Spread Functions of Compton Scatter Cameras*

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Abstract

A technique has been devised for independently determining the energy and spatial resolution components of the point spread functions of electronically collimated (Compton scatter) imaging devices using a Monte Carlo method. Monte Carlo runs are performed for the device being examined, and detailed records of exact energy loss and spatial coordinates of each interaction for each particle in the simulation are made. Taking resolution parameters from the known performance of the detectors, measured data is simulated by sampling from Gaussian distributions (for both energy and position) about the exact interaction energy and position in both detectors, for each Monte Carlo history. By back-projecting cones on an event-by-event basis and calculating the closest approach of the cone to the known source point using the simulated measurement data for one of the energy and position variables and the exact interaction data for the rest of the variables, the effect of the variable in question on the point spread can be isolated. In this paper we present the calculations of the contribution to the point spread function of the energy and spatial resolution in each detector for various source positions for a representative Compton camera configuration.

I. Introduction

Researchers have been attempting to develop electronically collimated (Compton scatter) nuclear medicine imaging devices since the pioneering work of Singh and coworkers in the early 1980's [1]. Such dual detector devices rely on the coincident detection of a Compton recoil electron (energy and position) and a Compton scattered photon, and the kinematic relationship between the energy of the electron and the photon scattering angle to predict a cone of potential source positions. The primary advantage such systems have over mechanically collimated devices is high efficiency, as the probability of the design event (a Compton scatter and a capture of the scattered photon) is much higher than the transmission of typical collimators. The main drawback is that the resolution

Figure 1: Schematic of a typical Compton scatter camera.

is degraded by uncertainties in each of several measurements (positions and energies) which are required for reconstruction. A schematic of a representative device and the basic basic back-projection methodology is shown in Figure 1. Though other types of reconstruction algorithms have been investigated for this type of device ([2, 3], the analysis of the present work requires only a simple back-projection. Since the position of both the initial Compton scatter (position (x_1, y_1, z_1) of the figure) and the final photo-absorption ((x_2, y_2, z_2)) are known, the direction of the gamma after the scatter can be determined. Further, since the energy of the Compton interaction has been measured, the angle of the scatter (θ_{KN}) can be determined from the Klein-Nishina formula. Since, however, all azimuthal angles are equally probable, the vector from the first interaction position back through the scattering angle determines not a line but rather sweeps a cone (at angle θ_{KN} from axis $\vec{A}$) of possible source positions. If thousands of such cones are constructed, they will intersect at the various source positions (in this work we will be concerned only with point sources).

Because of finite spatial and energy resolution of the component detectors, the cones projected using the measured data from each coincident event span a finite, rather than infinitesimal, solid angle of the source space, pro-

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Figure 2: Geometric sources of resolution degradation.

ducing a spread in predicted source positions about true positions, as shown in Figure 2. Clearly, the inexact position measurements produce misalignment of the calculated and actual scattered direction vector $\vec{A}$, and the finite energy resolution of the first detector leads to an error in the determination of θ_{KN}, and hence further misalignment of the computed and actual source cones. It is instructive to know the relative degree of the degradation in the total system resolution caused by these measurement errors, and we present here a Monte Carlo method for determining on a component-by-component basis the geometry and energy resolution constrained point spread function for Compton scatter cameras.

II. Methods

We chose a design using existing devices for the dual detector system in our investigation. The first (scatter) detector consists of a single crystal germanium detector, segmented into a 4 by 4 array of elements .5 cm on a side and .6 cm deep, as described in [4]. The capture detector is an annular single crystal of sodium iodide [5], 15 cm long, with an inner radius of 10 cm, and 1 cm thick. The schematic of Figure 1 describes this device well.

The performance of this device is modeled by Monte Carlo simulation. The transport program used here [6] employs a cross section data set similar to that used in TIGER, the photon simulation programs developed and supported by Los Alamos National Laboratories [7]. Photoelectric cross sections are treated on a subshell by subshell basis through 10 eV or the N shell (which is averaged) whichever is lower in energy, using the expanded parameterization of Biggs and Lighthill [8]. Carter and Cashwell [9] have suggested that the total photoelectric cross section can be resolved into individual subshell cross sections by assuming that the ratio between the cross sections for the n^{th} and $n^{th}+1$ levels is constant at all energies greater than E_n and is given by their ratio at E_n, and this scheme is employed throughout the entire energy range. Compton scattering is treated using a modification of the standard Klein-Nishina formula, taking into account the population of electrons in the atom and the probability that the electron will be ejected after having absorbed the photon momentum. This is done by multiplying σ_{KN} by the incoherent scattering function $S(Z,q)$, where q is related to the momentum transfer and is given by $q = 2\pi \sin(\theta/2)/\lambdabar$ if $\lambdabar$ is the reduced photon wavelength. Form factors for this work are taken from Hubbell *et al*[10]. Coherent scattering, here meant to include only elastic scattering from the bound atomic electrons (sometimes referred to as Rayleigh scattering) is also treated, and the cross section is given as the classically derived Thomson formula for the elastic scattering of a photon by a particle at rest times the square of the atomic form factor $F(Z,q)$. The form factors, like the scattering functions, are taken from tabulated data of Hubbell *et al.* Characteristic x-rays are generated (and subsequently tracked) on a subshell basis also, with all transitions from M shell inward treated explicitly, subject to the condition that the emitted x-ray have energy greater than the code cut-off (generally set to 1 keV). Data for the possible de-excitation transitions (energies and probabilities) is derived from that presented by Halbleib and Morel [11] and used in the P-codes (extended relaxation) of the ITS series.

The simulation as applied here departs from conventional treatments in that instead of tallying and printing a global summary of the simulation, the Monte Carlo program writes to disk the exact interaction position (the points (x_1, y_1, z_1) and (x_2, y_2, z_2) of Figure 1) and energy loss (E_1 in the same figure) incurred by every Compton scattered photon of the run. For multiply scattering gammas, a set of positions and energies were recorded. Typically, 10,000 photons are tracked and recorded in this manner, requiring very short run times and consuming less than a 100 kilobytes of disk to store the results. An isotropic, non-attenuating source was assumed, and for efficiency, incoming gammas were constrained to directions incident on the first detector. No variance reduction schemes where used.

This exact Monte Carlo data is then processed on a particle-by-particle basis by a back-projection program which analyzes each parameter of the event data and simulates its measurement. This is done by sampling from Gaussian distributions (using variances which characterize the actual detector performances) about the exact interaction energy and positions in both detectors, for each Monte Carlo history. This produces, for each gamma, a simulation not only of its physical history, but also of its measurement. For example, a gamma being absorbed in the second detector at (x_2, y_2, z_2) would have that interaction position smeared according to a Gaussian distribution with $FWHM$ given by the resolution of the second detector. It would then be this position used in examining the characteristics of the device. For gammas which multiply scatter, it is assumed that the interaction depositing the

Figure 3: Distance of closest approach.

most energy would be the one which was registered by the device.

Since the initial source position for the Monte Carlo simulation is known, it is possible using simple trigonometric manipulations to derive an expression for the closest distance of approach of the back-projected cone (derived from simulated measured data) to the source point (see Figure 3). These distances are calculated and tallied (in 1 mm wide bins) for every particle in the simulation, and the resulting distribution function can then be interpreted as the geometric point spread (psf_g) for the given detector and source position. We use the distribution of thicknesses of the back-projected cones to quantify the system because it is the one measure that is determined solely by the energy resolution of the first detector in concert with the spatial resolution of the first and second detectors. Other measures, based on reconstruction algorithms, introduce bias-variance tradeoffs which are not likely to be the same for different systems, and so preclude comparisons. The *FWHM* of the thickness distribution of the back-projected cones is the simplest parameter defining the "natural" or geometric performance of this system, and can be directly compared to similar characterizations of resolution for other imaging systems.

The method was tested by back-projecting cones derived from the exact data from the Monte Carlo (with no approximation for the measurements). As expected, a delta function for the point spread (within machine limits) was determined, *i.e.*, under the condition of perfect mesaurements, all cones pass exactly through the source.

We now apply this technique using combinations of exact data (for all variables but the parameter being studied) and simulated measured data, to isolate the effect of individual components of resolution degradation on the inherent (geometric) point spread function.

Run	Coh Scat 1st Det	Coh Scat 2nd Det	Mult Compt	Total False	True Events
1	.0794	.0487	.0515	.180	.777
2	.0851	.0469	.0517	.184	.771
3	.0819	.0468	.0504	.179	.774
4	.0990	.0456	.0547	.199	.748

Table 1: Breakdown of false coincident events (as fraction of all coincidences) for several source locations.

III. Results

Simulations were performed assuming a technetium energy point source at various distances and positions relative to the cylinder axis. Some pertinent results are summarized here. (Note, In all cases, efficiencies that are given refer to simulations at various source locations, while all psf_g results are for on-axis sources only.)

One predicted source of resolution blur in Compton cameras is the registering of false (but not random) coincidences due to coherent scattering in either detector and to dual Compton scattering in a single element of the scatter detector. In both of these situations, the measured direction of the scattered photon is skewed from the true direction, thus introducing error in the reconstruction. A summary of the frequency of such mis-registered events is given in Table 1 for several source positions (labeled runs 1-4 in the table). The 4 source conditions were: on axis at 10 cm from the detector face; on axis at 15 cm; off axis by 5 cm; and off axis by 10 cm. The results indicate that there is a large component of multiply scattered radiation recorded by this system. It is seen from the table, that the majority of multiple scatters were due to coherent scattering in the germanium detector, though there was also significant multiple Compton scatter and coherent scattering in the second detector.

It was found however that despite the frequent occurence (18%) of mis-registered coincidences (note, the sum of the false coincidences and true coincidences does not add up to 1.0 because roughly 4% of all coincidences can be rejected immediately as kinematically impossible, *i.e.*, total energy deposited is greater than the maximum energy transfer in a single Compton scatter), the effect on the psf_g is more to cast a very long low level tail in the point spread rather than produce a broadening in a peak. The plot of psf_g resembled a delta-function (within the bounds of the 1 mm source space pixels) with almost negligible noise.

Broadening in the psf_g due to finite energy resolution in the first detector is determined by assuming that the energy deposition in an experimental run of the first detector for a given event can be simulated in the Monte Carlo by sampling from a Gaussian distribution centered on the exact Monte Carlo energy deposition (E_{dep}) for each particle in the simulation. The variance of this Gaussian is derived

Figure 4: Point spread function due to energy resolution limitation.

Figure 5: Point spread function due to first detector lateral spatial resolution limitations.

from a full width at half maximum ($FWHM$) given by

$$FWHM = \left(1.064 \times 10^{-3} E_{dep} + 1.055\right)^{1/2},$$

which is a parametric fit to experimental data for the given germanium detector. The psf_g determination algorithm described earlier is then applied, using the exact Monte Carlo positions and the simulated energy deposition. A plot of the distribution of distances of closest approach of back-projected cones to the known source point is shown in Figure 4. The $FWHM$ of this distribution (assumed to be twice the half width at half maximum, as the psf_g is symmetric) in this case is roughly .45 cm for an on axis source 10 cm from the face of the front detector.

Note that there is no effect due to energy resolution uncertainties in the second detector because the energy deposited in there must fall within a window about $E_0 - E_{dep}^{1^{st}}$, (where E_0 is the source energy and $E_{dep}^{1^{st}}$ is the energy deposited in the first detector) in order for the coincidence to be accepted. The operation is binary: accept or reject, and the energy used in determining the scattering angle is taken from the measurement in the first detector.

The lack of all positional information other than which element fired in the first detector leads to the introduction of another component error. To determine the effect of approximating longitudinal positions, it was assumed originally that the z position of the interaction should be taken from the midpoint of the first detector. This lead to a $FWHM$ for the first detector longitudinal psf_g of a rather significant .45 cm. It was then realized that even though the majority of the initial interactions would take place near the front surface of the detector, the average interaction position from which a Compton scattered photon would escape the first detector would be deeper inside the detector than the midway point. Although such an average position is derivable for a given second detector configuration, as an initial approximation a distance of 4.5 mm (3/4 of the way through the detector) was assumed. Using this assumption a first detector longitudinal error $FWHM$ of roughly .32 cm was found.

The resolution loss due to use of non-exact lateral interaction position data is determined similarly, holding all interaction parameters exact except substituting the discrete element values of x and y, and determining the distribution of distances of closest approach of the back-projected cones to the known source position. From Figure 5, it is seen that this effect produces the created degradation in psf_g, contributing a $FWHM$ of roughly .71 cm.

Spatial resolution effects in the second detector are slightly more complicated to describe than for the discrete element first detector, because interaction positions are measured by the array of photo-multiplier tubes. In the system currently being modeled, the longitudinal (axial) resolution is taken to be 4 mm full width at half maximum, and in simulating the measurement, a Gaussian distribution centered at the true Monte Carlo z coordinate is sampled. We find a $FWHM$ in the spread in the back-projected source position due to this effect is roughly .24 cm.

There are two components to the lateral (x and y) resolution degradation of this sort of continuous second detector. The first is simply that due to error in the measurement of the radial (θ) position of the interaction. This is treated similarly to the way in which the longitudinal error is treated, in that a Gaussian with full width at half maximum of 4 mm is sampled and the measurement is simulated by adjusting θ, and hence x and y.

The second effect on the lateral (in the x and y sense) resolution is due to the lack of precise information about the depth of the initial interaction. For the simulations here, the depth is estimated by determining a rough entrance angle from the measured z position and the fixed r_{inner} of the cylinder, and then assuming that the particle traveled approximately one mean free path before interacting, as seen in Figure 6.

Using this rather involved technique for assigning a mea-

Figure 6: Second detector penetration spatial measurement.

Figure 7: Point spread function due to second detector lateral (penetration component) spatial resolution limitation.

sured penetration depth to each interaction of the simulation could be done easily in an actual reconstruction algorithm by using a penetration depth vs. Z curve for the second detector). The psf_g's of the back-projection due to the penetration uncertainty in the detector lateral spatial resolution is shown in Figure 7 and displays a $FWHM$ of about 1 mm, for an on axis source at 10 cm. The error due to the radial component was shown to be negligible.

IV. Discussion

The contribution to the system resolution of the elements described above as determined by the methods here are summarized in Table 2.

The overall resolution for a detector of this configuration for on axis point sources at 10 cm is seen to be on the order of 9 mm, with the most significant contribution to that coming from errors caused by the poor lateral resolution in the first detector. The final entry, the overall $FWHM$,

Effect	$FWHM$ (cm)
Registration of Skewed Coincidences	0.00
Energy Resolution of First Detector	0.45
Lateral Resolution of First Detector	0.71
Longitudinal Resolution of First Detector	0.32
Lateral Resolution of Second Detector	0.10
Longitudinal Resolution of Second Detector	0.24
Overall $FWHM$	0.90

Table 2: $FWHM$ due to various components of detector uncertainty.

Case	Source Eff	Output γ's per input γ	2nd Det Eff	Tot. Eff. ($\times 10^4$)
1	.00484	.0741	.526	1.89
2	.00217	.0751	.521	0.85
3	.00299	.0724	.526	1.14
4	.00155	.0714	.543	0.60

Table 3: Photon efficiencies for various source locations.

was determined by using the simulated measurement data for all 7 of the interaction variables (scattering energy and two sets of positions). Note that the component $FWHM$'s add in quadrature to roughly the equivalent of the overall value, though it is not absolutely clear that they must in theory.

Thus the method has proven useful in quantifying the sources of the limitations in the geometric resolution of the device. Clearly, any future design must improve upon the choice of the first detector, particularly the spatial sensitivity. It is also clear that the selection for the second detector in this system was sound, contributing very little to performance degradation.

Finally, as the entire promise of electronically collimated cameras lies in expected gains in effeciency, we present computations of absolute efficiency for the current device for the 4 source cases in Table 3. Efficiencies have been broken into 3 components: simple solid angle subtended by the rather small first detector; the number of emerging Compton scattered photons per photon incident on the first detector (note, these could include multiply scattered gammas); and the probability of absorbing the emitted photons. This last factor takes into account the large number of low angle scatters which pass through the open back end of the hollow cylinder of the second detector, hence its somewhat low value. The efficiency for absorbing gammas which strike the detector is nearly 100%. Note that the numbers here include the 23% of coincidences which are actually mis-registered or kinematically impossible, and so the final efficiency should be adjusted accordingly.

The values for the overall sensitivity are somewhat misleadingly low, due exclusively to the small size of the first detector modeled here (this manifests itself in column 1, the measure of the source efficiency). Recalling that the design detector (selected for this analysis because the device exists and has been made available for performing measurements) is only 4 cm^2 in area, one can easily imagine a larger, and hence more efficient detector. For example, an 8 cm by 8 cm detector, would have, assuming that the sensitivity scales roughly linearly with area, an absolute efficiency of around 16 times that modeled above, detecting as designed Compton camera events up to .3% of all photons escaping the source volume.

V. Conclusions

A method has been devised for determining the impact on overall geometric point spread function of the various system components for Compton scatter cameras, and has been applied to a representative design. Major sources of resolution have been identified, permitting optimization guides for subsequent designs.

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